FIRST LESSONS IN MENTAL AND WRITTEN ARITHMETIC

ADDITION + SUBTRACTION −

MULTIPLICATION × DIVISION ÷

ILLUSTRATIONS BY ED SEARS.

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ROBINSON'S MATHEMATICAL SERIES.

FIRST LESSONS

IN

MENTAL AND WRITTEN

ARITHMETIC.

ON THE OBJECTIVE METHOD.

EDITED BY

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PREFACE.

THE author calls attention to the following points as among the claims made for this book.

1. It treats Number *objectively,* and by a method believed to be new and simple.

2. The principles and processes are unfolded in *natural order*, as occasion demands them.

3. The greatest pains have been taken in the attempt to present clearly the most elementary and fundamental principles, in the firm belief that no sound scholarship can be erected on any other basis, and that the elements are not only the most important but are also the most difficult to teach successfully. If a pupil ever needs guidance and help it is when he sets out in the path of knowledge. Therefore, the aim has been so to instruct and encourage him in his *first efforts* that, gaining strength at each step, he shall advance with increasing interest and delight.

4. Special attention has been devoted to Notation and Numeration. Numbers consisting of three periods have been developed objectively. It is believed that children can use numbers having nine figures as readily and intelligently as those having three figures, if Notation and Numeration be understood. A careful examination of the method used is earnestly solicited.

5. Throughout the book the pupil is taught to use his reason and common-sense, and, as a rule, is not required to memorize until he perceives the truth of what is to be committed to memory.

6. From the commencement mental and written work are combined. At the outset the pupil is set to working on the blackboard. He first obtains his results mentally, and afterwards reproduces his work with his hands, and looks upon it with his eyes. It then becomes to him a reality. What is traced by the muscles and pictured on the eye is the more indelibly imprinted

on the memory. It is for this reason that the book contains so much work in the form of Equations, with the work so varied and so full. It is believed that the pupil will thoroughly master the Tables, learning readily and accurately to perform all the elementary operations, by this process sooner than by any other; since he must use the Tables at every step.

7. Addition and Subtraction are treated in immediate connection; also Multiplication and Division. Thus their correlations are more clearly shown. Multiplication is at first worked up under Graded Addition, and Division under Graded Subtraction, till they are well understood, and Tables have been made and used, before the terms Multiplication and Division are given. In Division the three terms are written in precisely the same order as the corresponding terms in Multiplication, that Long Multiplication may illuminate and illustrate Long Division.

8. The Tables for Addition, Subtraction, Multiplication and Division are developed *objectively* and *progressively*.

9. The principles of Factoring have been so applied as to simplify Division and pave the way to Fractions.

10. The rule for Addition and Subtraction of Fractions having unlike Denominators is developed *objectively*.

11. Names, Definitions and Rules are given with the full conviction that the best time to give them is when we give the things *named* or *defined*, and the method of operation *described in a Rule*. If a child *cannot comprehend a method of operation*, he should *not be required to use it;* but if he *can*, then he *can comprehend a clear statement of the method in a Rule;* and *since he understands the Rule, and it is important*, HE SHOULD LEARN THE RULE.

The aim has been not to load down the pupil with Arithmetic as *a burden from without*, but to cause it to spring up *within him* by a natural and healthful process; that, growing and unfolding with his intellect, it may be an organized, vital and indestructible *part of himself.*

The author has written this book on his own plan, using his own methods and illustrations, striving to adapt them to the comprehension of children. He is not, however, so vain as to presume that he has made no mistakes, and has produced a work above criticism.

S. D. B.

June 1, 1870.

SUGGESTIONS TO TEACHERS.

The author of this book requests most earnestly that teachers will seek fully to comprehend his plan and methods, and strive to work by them, teaching the book precisely as it is written.

It is not taken for granted that the pupil knows anything about Number. The attempt is made to teach each thing in its proper place, accompanied by such reasons and explanations that the pupil shall *comprehend the subject*, and not be required *merely to memorize words and Tables to him almost without meaning*. The work is begun at the very basis, in the belief that the first and most important work is to lay broad foundation-stones, so firmly that, whoever shall build upon them, he shall build in confidence, knowing that he is on the *solid rock*.

The aim is so to instruct the pupil at the commencement that he shall be able to use the knowledge gained. The first few Lessons are quite simple, since he is to be put to working on the blackboard at once. When each figure and sign is first given he should be drilled in making it on the blackboard till he can write it neatly and with facility.

The Signs +, —, =, and the Equation by Addition and that by Subtraction, are carefully developed, with the full meaning and power of each item, so that the pupil shall at once be able to use them understandingly. By their use he will be able to write on the blackboard from day to day all he learns of Arithmetic, and to reproduce it in every possible form. Pupils love blackboard-work; and in no other manner can they be taught so successfully.

Particular attention should be given to the relation between Addition and Subtraction as shown in Lessons IX, X, XI, XII and XLIX.

The method of developing the Tables for Addition and Subtraction, and those for Multiplication and Division, by the use of cubes, and the manner of reading the Tables from the cubes, should be carefully explained.

The method of adding columns of numbers by 6's, shown on page 27, really embraces the substance of Addition. The pupil should be well drilled also in adding by 7's, 8's, 9's and 10's. This will lay a good foundation for subsequent work in Notation.

The greatest possible pains should be taken in the development of Notation and Numeration, as set forth on pages 46, 47, 62, 68, 74–76.

Addition by objects, as shown on pages 52 and 53, and Subtraction by objects, as given on pages 56 and 57, should be explained till every pupil can point out every step.

The Lessons on Graded Addition and Subtraction, commencing on page 81, are specially important, since they lay the foundation for Multiplication and Division, and enable the pupil to see that what follows is but a new application of what he has already learned.

For each of the numbers ***2*** and ***3*** two Multiplication Tables are given; but for each of the numbers ***4, 5, 6, 7, 8, 9, 10,*** only one Multiplication Table is given. In each case a second Table should be written out by each pupil, in the manner explained in Lesson LXIV, and be learned with the one given.

The Examples and Explanations showing the precise manner in which Multiplication is derived from Addition, and Division from Subtraction, should be dwelt upon till every pupil can solve the same Example both by Addition and Multiplication, or by Subtraction and Division.

Great pains should be taken to show clearly the relations between Multiplication and Division by the Solutions and Explanations given in the book. The *corresponding terms* in Multiplication and Division are *written in the same order*, that the pupil may the more readily understand the relations.

The difference between the two cases in Division given on pages 96 and 100 should be carefully pointed out.

The subject of Factoring as applied to Division, especially for the purpose of deducing the General Principles of Division, should receive special attention, since these Principles are important in Fractions.

It is left to the Teacher to give all needed Explanations in Denominate Numbers, and to supply any further Examples needed.

THE AUTHOR.

LESSON I.

1. In this picture, how many girls are in the swing?

2. How many girls are pulling the swing?

3. If you count both girls together, how many are they?

One girl and ***one*** other girl are how many?

4. How many kittens do you see on the stump?

5. How many on the ground?

6. How many kittens are in the picture?

One kitten and one other kitten are how many?

7. If you should ask me how many girls are in the swing, or how many kittens are on the stump, I could answer aloud, *One;* or I could write *One;* or thus, ***1.***

8. If I write *One,* this is called the *word One.*

9. This, ***1,*** is named a ***figure One,*** because it means the same as the word *One,* and stands for *One.*

10. Write 1. What is this named? Why?

11. A figure one may stand for *one* girl, *one* kitten, or *one* any thing.

12. When children first attend school, what do they begin to learn? *Ans.* Letters and words.

13. Could you read or write before you had learned either letters or words?

14. If we have all the *letters* together, they are named the Alphabet.

15. If we write or speak *words,* they are named Language.

16. You are commencing to study Arithmetic; and you can read and write in Arithmetic only as you learn the Alphabet and Language of Arithmetic. But little time will be required for this purpose.

LESSON II.

1. If we speak or write words, what do we name them, when taken together?

2. What are you commencing to study? *Ans.* Arithmetic.

3. What Language must you now learn?

4. What do we name this, 1? Why?

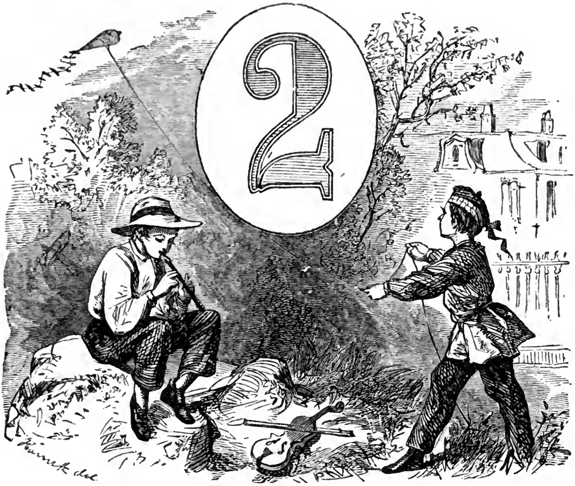
5. This figure, 1, is part of the Language of Arithmetic.

6. If I should write something to stand for ***Two***—*two* girls, *two* kittens, or *two* things of any kind, what do you think we would name it?

7. A ***figure Two*** is written thus: **2.** Make a *figure two.*

8. Why do we name this a *figure two?*

9. This figure two (2) is part of the Language of Arithmetic.



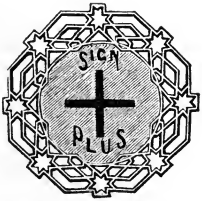
10. In this picture one boy is sitting, playing a flageolet. What is the other boy doing? If the boy standing should sit down by the other, how many boys would be sitting together? One boy and one other boy are how many boys?

11. You see a flageolet and a violin. They are musical instruments. One musical instrument and one other musical instrument are how many?

12. I will write thus: 1 1 2. We say that 1 boy and 1 other boy, counted together, are 2 boys; or are equal to 2 boys. We will now write something to show that the first 1 and the other 1 are to be counted together.

13. We name a line drawn thus, —, a ***horizontal line.*** Draw such a line. Name it.

14. A line drawn thus, |, we name a ***vertical line.*** Draw such a line. Name it.



15. Now I will put two such lines together; thus, +. What kind of a line do we name the first (—)? And what do we name the last (|)? Are these lines long or short? Where do they cross each other?

16. Each of you write thus: —, |, +.

17. This, +, is named ***Plus.*** *Plus* means *more;* and + also means *more.*

18. I will write,

One and One More Equal Two.

19. Now I will write part of this in the Language of Arithmetic. I write the first *One* thus, 1; then the other *One* thus, 1. Afterward I write, for the word *More,* thus, +, placing the + between 1 and 1, so that the whole stands thus: 1 + 1. As I write, I say, *One and One more.*

20. Each of you write 1 + 1. Read what you have written.

21. This +, when written between the 1's, shows that they are to be put together, or counted together, so as to make 2.

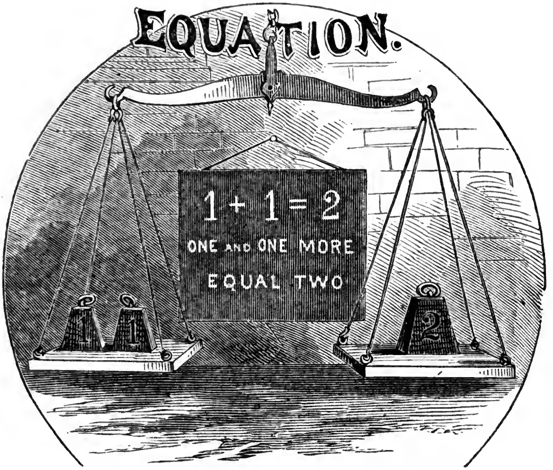
22. Because + shows what is to be done, it is called a *Sign.* If we take its name, *Plus,* and the word *Sign,* and put both words together, we have *Sign Plus,* or *Plus Sign.* In speaking of this we may call it *Sign Plus,* or *Plus Sign,* or *Plus.*

23. 1, 2, +, are part of the Language of Arithmetic.

Write the following in the Language of Arithmetic:

24. One and one more. *25.* One and two more.

26. Two and one more.



LESSON III.

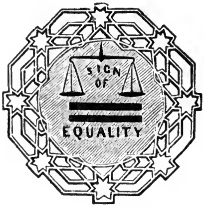
ONE AND ONE MORE EQUAL TWO.

1. I will write the above thus: 1 + 1 equal 2.

2. In length, are these lines, =, *equal* or *unequal?*

3. We will use two lines thus drawn, =, to mean *equal,* in place of the word *equal.* Writing them in 1 + 1 equal 2, we have 1 + 1 = 2.

4. Because these two lines thus written show something, they are called a *Sign.* And because they mean *equal,* we name them the ***Sign of Equality.***



5. Anything written that means something, as, for instance, 1 + 1 = 2, is called an ***Expression.***

6. An expression like the above, in which we use the Sign of Equality, is named an ***Equation.***

2

LESSON IV.

1. How many boys are in this boat?

2. If the boy in the end of the boat should fall out, how many boys would be left in the boat? *One* boy from *two* boys leaves how many boys?

3. Instead of saying, *One boy from two boys,* we will say, *Two boys less one boy,* which means the same. Then, two boys less one boy will equal how many?

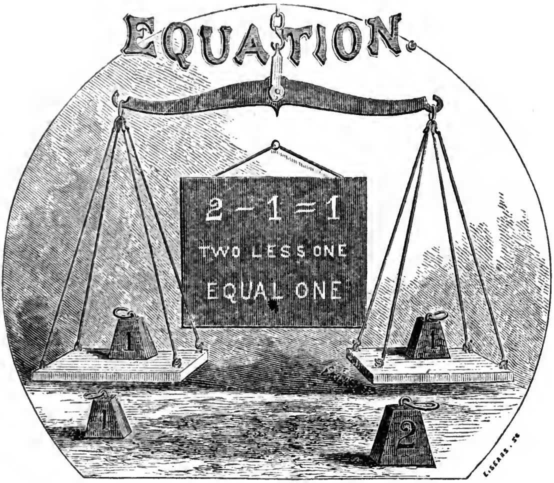
4. We will write, 2 less 1 = 1. Read this.

5. We wish something that means *less,* to write between 2 and 1, for the word *less.*

6. A line written thus, —, is used to mean *less,* instead of the word *less.*

7. The name of this line, —, when thus used, is ***Minus.*** *Minus* means *less.*

8. We may write 2 less 1 = 1, or 2 — 1 = 1.



9. Since this, —, shows that something is to be done, what may we call it? Ans. A Sign.

What is its name?

10. If we put the two words *Sign* and *Minus* together, they make *Sign Minus,* or *Minus Sign.* In speaking of this Sign we may call it *Sign Minus,* or *Minus Sign,* or *Minus.*

11. Each of you write, 2 − 1 = 1. Read aloud what you have written.

12. What is the name of this Expression: 2 − 1 = 1?

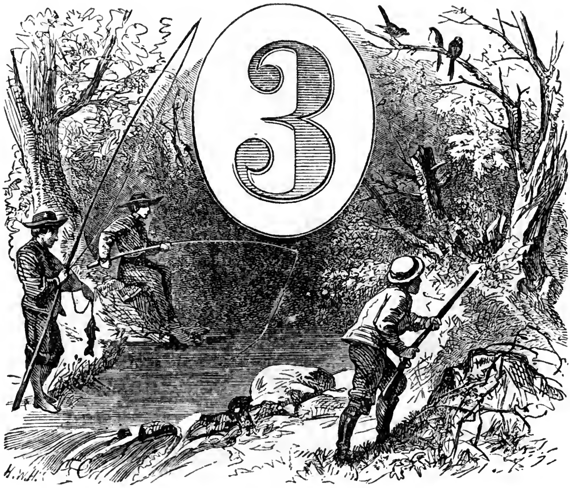
13. Write, and name in order, the following: 1, 2, +, =, —. Of what do these form part?

Write, in Arithmetical Language, the following:

14. One equals one.

15. One and one more equal two.

16. Two less one equal one.



LESSON V.

1. In this picture how many boys do you see fishing? How many hunting? How many in all? 2 boys and 1 other boy are how many?

2. We make a ***figure Three*** thus: ***3.***

3. 1 boy and 2 more are how many boys? Are 1 boy and 2 more just as many as 2 boys and 1 more?

4. How many birds are on this tree?

5. If 1 of them should fly away, how many would be left? 1 bird from 3 birds leaves how many birds?

6. If, instead, 2 of the birds should fly away, how many would be left? 3 birds less 2 birds are how many?

7. 1 from 3 leaves how many? 2 from 3?

8. 3 boys are how many more than 2 boys? Than 1?

9. Write, and read aloud, the following Equations:

$1 + 1 = 2$; $2 - 1 = 1$; $2 + 1 = 3$;

$1 + 2 = 3$; $3 - 1 = 2$; $3 - 2 = 1$.

LESSON VI.

1. In this picture you see 1 driver? How many other men in the wagon? 1 man and 3 other men are how many? 3 men and 1 other man are how many?

2. We write a ***figure Four*** thus: ***4.***

3. Are 3 men and 1 man more just as many as 1 man and 3 men more?

4. If the driver should jump from the wagon, how many men would be left in the wagon? 1 man from 4 men leaves how many men?

5. If, instead, the 3 other men should jump out, how many would be left in the wagon? 3 men from 4 men leave how many men?

6. 4 men are how many more than 3? Than 1 man?

7. How many horses are in 1 span?

8. How many spans of horses are drawing this wagon?

9. How many horses are there in all?

10. 2 horses and 2 other horses are how many horses?

11. If the 2 horses in front should be unhitched and driven away, how many would be left? 2 horses from 4 horses leave how many horses?

12. 4 horses are how many more than 2 horses? How many more than 3 horses? Than 1 horse?

13. 3 horses and 1 horse are how many? 1 horse and 3 horses? 2 horses and 2 other horses?

14. 1 horse from 4 horses leaves how many? 3 horses from 4 leave how many? 2 from 4?

15. 2 horses are how many less than 4 horses?

16. 1 horse is how many less than 4 horses?

17. What Language have we commenced learning?

18. Write these: 1, 3, 2, 4, +, –, =. Name each.

19. Of what Language are they part?

Write, in Arithmetical Language, the following:

20. One and one more equal two ($1 + 1 = 2$);

21. Two less one equal one;

22. Two and one more equal three;

23. One and two more equal three;

24. Three less one equal two;

25. Three less two equal one;

26. Three and one more equal four;

27. One and three more equal four;

28. Two and two more equal four;

29. Four less one equal three;

30. Four less three equal one;

31. Four less two equal two;

32. The Sign + shows that what is written at the right of it is to be *counted with* what is written before it.

33. The Sign – shows that what is written at the right of it is to be *taken away from* what is written before it.

LESSON VII.

1. In this picture, Minnie has 1 rose in her left hand; how many has she in her right hand?

2. If she should put them all in her right hand, how many would she have in her right hand?

3. We make a ***figure Five*** thus: ***5.*** Make one.

4. 4 roses and 1 rose more are how many roses? 1 rose and 4 more roses are how many?

5. Are 4 roses and 1 more just as many as 1 and 4 more?

6. Willie has 3 roses in his right hand; how many has he in his left hand? If he should put them all in his right hand, how many would he then have in his right hand? 3 roses and 2 roses are how many? 2 roses and 3 roses are how many?

7. Are 3 roses and 2 more just as many as 2 and 3 more?

8. If Minnie should give her teacher the rose in her left hand, how many would she have left? 1 rose from 5 roses leaves how many roses?

9. If, instead, she should give away the 4 roses in her right hand, how many would she have left? 4 roses from 5 roses leave how many?

10. If Willie should give his mother the 2 roses in his left hand, how many would he have left? 2 roses from 5 roses leave how many?

11. If, instead, he should give her the 3 roses in his right hand, how many would he have left? 3 roses from 5 roses leave how many?

12. How many more roses has Willie in his right hand than in his left? 3 are how many more than 2?

13. How many more has Willie in his right hand than Minnie in her left? 3 are how many more than 1?

14. How many more roses are in Minnie's right hand than in Willie's? 4 are how many more than 3?

15. How many more has she in her right hand than Willie in his left? 4 are how many more than 2?

16. How many more has she in her right hand than in her left? 4 are how many more than 1?

17. How many roses are on the rose-bush?

18. How many would be left if Minnie should pick 1? If she should pick 4? If 2? If 3?

19. 5 are how many more than 4? Than 1? Than 3? Than 2?

Write, in Arithmetical Language, the following:

4 and 1 more equal 5;	5 less 1 equal 4;
3 and 2 more equal 5;	5 less 2 equal 3;
2 and 3 more equal 5;	5 less 3 equal 2;
1 and 4 more equal 5;	5 less 4 equal 1.

LESSON VIII.

1. How many chickens are on this hen's back?

2. How many other chickens are about her?

3. 5 chickens and 1 chicken are how many?

4. We make a ***figure Six*** thus: ***6.*** Make one.

5. How many ducklings are swimming in this stream?

6. How many are on shore?

7. How many ducklings in all? 4 ducklings and 2 more are how many? 2 and 4 more are how many?

8. How many swallows are on this gate?

9. How many others are on the fence?

10. 3 swallows and 3 other swallows are how many?

11. If a hawk should fly away with the chicken on the hen's back, how many chickens would be left? 1 chicken from 6 chickens leaves how many?

12. If a fox should steal the 2 ducklings on shore, how many ducklings would be left?

13. If, instead, the 4 ducklings should float away, how many would be left? 2 ducklings from 6 ducklings leave how many? 4 from 6 leave how many?

14. If the 3 swallows on the gate should fly away, how many would be left? 3 swallows from 6 leave how many?

Write the following in Equations:

5 and 1 more equal 6;	6 less 1 equal 5;
4 and 2 more equal 6;	6 less 2 equal 4;
3 and 3 more equal 6;	6 less 3 equal 3;
2 and 4 more equal 6;	6 less 4 equal 2;
1 and 5 more equal 6;	6 less 5 equal 1.

15. Read aloud, in concert, what you have written.

16. James had 3 pennies, and his father gave him 2 more; how many had he then? He found 1 more; how many had he in all?

17. Flora had 4 nice dresses for her doll, and her mother made 2 new ones for it; how many dresses for her doll had she then?

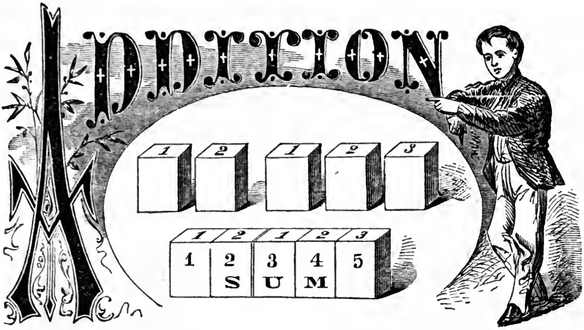
18. Charlie caught 3 trout, and Willie 3; how many did both catch?

19. Mary had 2 canaries that sang, and 4 that were not singers; how many canaries had she in all?

20. Albert had 6 pennies, and gave 3 of them for 6 apples; how many pennies had he left? He ate 2 of the apples; how many had he left? His sister Helen ate 2 more of them; how many, in all, did Albert and Helen eat? How many apples had Albert then left?

21. 4 and 1 are how many? 2 and 3? 4 and 2? 3 and 3? 1 and 5? 2 and 4? 3 and 2? 5 and 1?

22. 2 from 5 leave how many? 3 from 5? 2 from 6? 4 from 6? 3 from 6? 1 from 6? 5 from 6?



LESSON IX.

1. One, Two, Three, &c., are called ***Numbers;*** and because the figures 1, 2, 3, &c., stand for these numbers, they are themselves commonly called numbers.

2. When we put 2 and 3 together, or unite them, and find that they equal 5, we are said to ***Add*** them.

3. Uniting two or more numbers, and finding what number they equal, when taken together, is named ***Addition.***

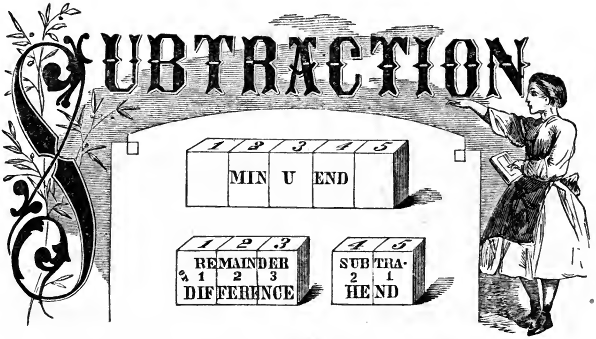
4. When numbers are to be *added,* we usually write them in a vertical column. Let us add 2, 3, and 1. Writing the numbers as shown in the margin, and drawing a line below the column, we first find that 1 and 3 more equal 4. Next we add this 4 and the remaining number, 2, and, finding that 4 and 2 more equal 6, we write 6 below the line. 6 is named the ***Amount,*** or ***Sum,*** of 2, 3, and 1.

$$\begin{array}{r} 2 \\ 3 \\ \underline{1} \\ 6 \end{array}$$

Find and write the Sum in each of the following

EXERCISES FOR THE SLATE AND BLACKBOARD.

1	3	2	1	4	1	3	2	2	2	2	1	3
2	1	2	4	1	3	2	3	2	1	1	3	1
1	1	1	1	1	2	1	1	2	2	3	1	2
—	—	—	—	—	—	—	—	—	—	—	—	—



LESSON X.

1. When we take 2 from 5 and find that 3 remain, or are left, we are said to *Subtract* 2 from 5. *Subtract* means *take away.*

2. Taking one number from another, and finding how many remain, is named ***Subtraction.***

3. Since 5 is *diminished,* or *made less,* by subtracting 2 from it, we name 5 the ***Minuend.*** *Minuend* means *to be diminished.*

4. Since 2 is *subtracted* from 5, 2 is named the ***Subtrahend.*** *Subtrahend* means *to be subtracted.*

5. Since 3 shows how many *remain* after subtracting 2 from 5, we name 3 the ***Remainder,*** or the ***Difference*** between 5 and 2.

6. In performing Subtraction we usually write the work in the form shown at the right hand.

SUBTRACTION.

5 *Minuend.*
2 *Subtrahend.*
3 *Remainder.*

Exercises for the Slate and Blackboard.

5	3	4	4	4	5	5	5	6	6	6	6	6
3	1	3	1	2	4	1	2	5	1	4	2	3
2												

LESSON XI.

$2 + 3 = 5.$

1. The Sign of Equality divides every Equation into two parts, named ***Members.***

2. The *First Member* and the *Second Member* of every Equation are equal, and the Sign of Equality stands between them.

3. Since the Equation $2 + 3 = 5$ is formed by *Addition,* we name it an ***Equation by Addition.***

4. In every Equation by Addition like the above, having *three* numbers, with the *greatest* standing *last,* if only one of the numbers be missing it is easy to find it.

Since the Sum of the first two numbers equals the third, it is evident that if either of them be subtracted from the third the Remainder will equal the other number.

We may find any one of the three numbers thus:

Missing Numbers.	Methods of Finding.
FIRST.	*Subtract* the SECOND from the THIRD.
SECOND.	*Subtract* the FIRST from the THIRD.
THIRD.	*Add* the FIRST and SECOND.

Write the proper numbers in place of (?) in these

EXERCISES FOR THE SLATE AND BOARD.

2 + 3 = ?	3 + 1 = ?	? + 1 = 5	? + 3 = 4
2 + 2 = ?	3 + 2 = ?	4 + ? = 5	2 + ? = 3
1 + 4 = ?	3 + ? = 5	2 + ? = 4	1 + ? = 3
1 + 3 = ?	? + 3 = 5	3 + ? = 4	? + 2 = 3
2 + 1 = ?	1 + ? = 5	? + 2 = 4	2 + 4 = ?
2 + ? = 5	? + 4 = 5	? + 1 = 4	3 + ? = 6
3 + 3 = ?	4 + ? = 6	1 + ? = 6	? + 4 = 6

LESSON XII.

$5 - 2 = 3.$

1. Since the Equation $5 - 2 = 3$ is formed by *Subtraction,* we will name it an ***Equation by Subtraction.***

2. In this, and in every Equation by Subtraction having only three numbers, and the largest of the numbers standing first, the first number is equal to the Sum of the two others.

3. If the second number be subtracted from the first, the Difference will equal the third; and if the third be subtracted from the first, the Difference will equal the second.

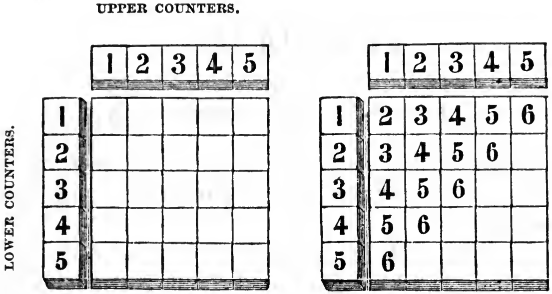
From this it is evident that in any such Equation by Subtraction, we may find any one of the numbers thus:

Missing Numbers.	Methods of Finding.
FIRST.	*Add* the SECOND and THIRD.
SECOND.	*Subtract* the THIRD from the FIRST.
THIRD.	*Subtract* the SECOND from the FIRST.

Find and write the missing numbers in the following

EXERCISES FOR THE SLATE AND BOARD.

$5 - 2 = ?$	$5 - ? = 4$	$? - 1 = 3$	$3 - 1 = ?$
$5 - 4 = ?$	$? - 3 = 2$	$4 - 1 = ?$	$3 - ? = 1$
$5 - ? = 3$	$? - 2 = 2$	$4 - 3 = ?$	$3 - 2 = ?$
$5 - ? = 1$	$? - 2 = 3$	$4 - 2 = ?$	$3 - ? = 2$
$5 - 3 = ?$	$? - 4 = 1$	$4 - ? = 2$	$6 - 2 = ?$
$5 - 1 = ?$	$? - 1 = 4$	$4 - ? = 1$	$6 - ? = 3$
$5 - ? = 2$	$? - 3 = 1$	$4 - ? = 3$	$? - 2 = 4$
$6 - 3 = ?$	$6 - 4 = ?$	$? - 3 = 3$	$6 - ? = 4$
$? - 4 = 2$	$6 - 1 = ?$	$6 - 5 = ?$	$6 - ? = 2$



LESSON XIII.

We now proceed to make a Table, on the plan shown above. The Table will help us in adding and subtracting, in performing the work in all Examples such as we have had.

First, we arrange, side by side, 5 rows of little cubes, with 5 cubes in each row. Then we place another row of 5 cubes a little above these, and a like row a little to the left of the 5 rows. We number and write on the cubes of the last two rows, "1, 2, 3, 4, 5," as shown. These two rows, thus numbered, are to be used as Counters.

Suppose we wish to add 2 and 3 more, and write the Sum in the Table. We take from the Lower Counters the cubes numbered "1, 2," and from the Upper Counters the cubes numbered "1, 2, 3," and, adding them, find their Sum is 5. We then write this Sum on the cube which stands at the right of the cube numbered "2" in the Lower Counters, and below the cube numbered "3" in the Upper Counters.

In the same manner we find and write the Sum of any other two numbers written on the Counters.

The Table is filled as far as 6. We read it thus:

1 and 1 are 2,	2 and 1 are 3,	3 and 2 are 5,
1 and 2 are 3,	2 and 2 are 4,	3 and 3 are 6;
1 and 3 are 4,	2 and 3 are 5,	4 and 1 are 5,
1 and 4 are 5,	2 and 4 are 6;	4 and 2 are 6;
1 and 5 are 6;	3 and 1 are 4,	5 and 1 are 6.

We will name this an ***Addition Table.*** We may also use it as a ***Subtraction Table.***

If we *add* 2 and 3, their *Sum* is 5; as appears in the Table. If we *Subtract* 2 from 5, the *Difference* is 3. We obtain this result from the Table, thus: 1st, we find in the Lower Counters 2, which is to be subtracted; 2d, we pass from this 2 along to the right, and find 5, from which 2 is to be subtracted; 3d, directly above this 5, in the Upper Counters, we find the 3, which is the Difference between 2 and 5.

We read this as a Subtraction Table thus:

1 from 2 leaves 1,	2 from 3 leave 1,	3 from 5 leave 2,
1 from 3 leaves 2,	2 from 4 leave 2,	3 from 6 leave 3;
1 from 4 leaves 3,	2 from 5 leave 3,	4 from 5 leave 1,
1 from 5 leaves 4,	2 from 6 leave 4;	4 from 6 leave 2;
1 from 6 leaves 5;	3 from 4 leave 1,	5 from 6 leave 1.

Exercises for the Slate and Board.

$2 + 3 = ?$	$1 + ? = 6$	$4 - 1 = ?$	$3 + ? = 6$
$2 + 4 = ?$	$4 + ? = 6$	$3 - 2 = ?$	$1 + 4 = ?$
$3 - 1 = ?$	$2 + ? = 6$	$1 + 3 = ?$	$5 + ? = 6$
$4 + ? = 5$	$1 + 5 = ?$	$5 - ? = 2$	$4 - 2 = ?$

Written Exercises.

1. Frank had 4 peaches, and Henry 2; how many had both boys?

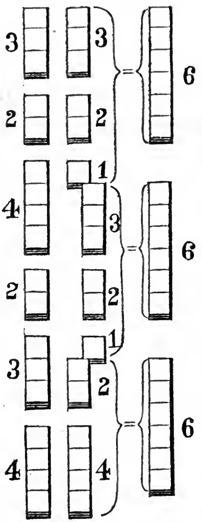
2. Emma had 6 pinks, and gave 3 of them to Walter; how many had she left?

LESSON XIV.

We will now add 3, 2, 4, 2, 3 and 4, as shown at the right hand, and find how many times we can make the number 6 from them.

Beginning at the bottom, the first number is 4. From the 3 next above this we take 2, which, added to 4, make 6. Having taken 2 from 3, we have 1 left. We now add this 1 to the 2 standing above the 3, and have 3 for the Sum. Taking 3 of the 4 ones next above, and adding them to this Sum, 3, we have 6. Next we add the 1, left from 4, to the 2 standing above the 4, and find that their Sum is 3. Adding together this Sum, 3, and the last number, 3, we find their Sum to be 6. Thus we make the number 6 *three times* from the whole column. We write each 6 in the Sum, and write + between the 6's.

By Objects.



By Figures.

$$\begin{array}{l}3 = 3\\ 2 = 2\\ 4 = \begin{cases}1\\3\end{cases}\\ 2 = 2\\ 3 = \begin{cases}1\\2\end{cases}\\ 4 = 4\end{array}\qquad \begin{array}{l}=6\\ \\ =6\\ \\ =6\end{array}$$

6+6+6. Sum.
or
3 times 6

While performing the work we say thus: 4 and 2 are 6; 2 from 3 leave 1, 1 and 2 are 3, 3 and 3 are 6; 3 from 4 leave 1, 1 and 2 are 3, 3 and 3 are 6. The entire column is equal to 3 times 6.

In the same manner perform the work in each of these

EXAMPLES FOR THE SLATE AND BOARD.

1	4	2	2	3	3	5	4	4	1	5	4	5
4	1	3	2	5	5	5	4	3	3	5	5	4
2	4	2	2	4	2	5	4	3	5	2	5	5
3	2	4	4	3	1	5	4	3	4	3	3	3
5	5	3	4	2	4	5	4	2	3	4	3	4
3	2	4	4	1	3	5	4	3	2	5	4	3

LESSON XV.

1. In this scnool, one boy has 6 books on his desk, and another has 1; how many books have both on their desks? 6 books and 1 book are how many?

2. We make a ***figure Seven,*** thus: *7*. Make one.

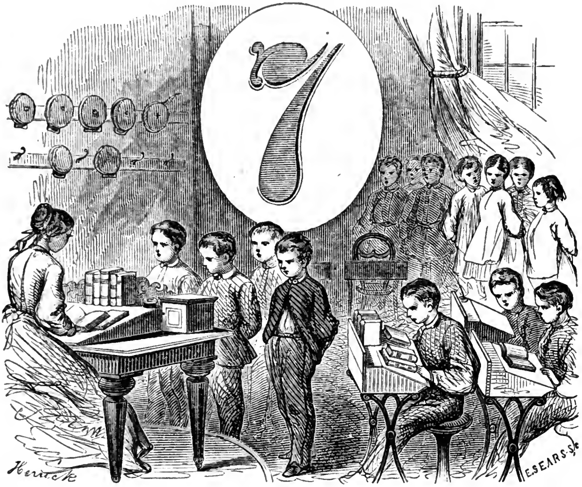
3. How many caps are hanging in the upper row? How many in the lower row? How many caps are there in all? 5 caps and 2 caps are how many?

4. How many boys are standing? How many are sitting? How many boys are there in all? 4 boys and 3 boys are how many?

5. How many girls in this school? If 1 of them should go home, how many would be left? 1 girl from 7 girls leaves how many?

6. If 2 of the 7 boys should go and take their caps and go home, how many boys would be left? 2 boys from 7 boys leave how many? 2 caps from 7 caps leave how many?

7. How many girls are standing? If 3 of them should sit down, how many would be left standing?



If the girls so left standing should go home, how many girls would be left? 4 girls from 7 girls leave how many? 3 girls from 7 leave how many?

8. How many books are on the teacher's desk? If 5 of them should be taken away, how many would be left? 5 books from 7 leave how many?

WRITTEN EXERCISES.

1. James had 3 marbles, and Harry gave him 4 more; how many had he then? He lost 2 marbles; how many had he left?

2. Charlie had 2 peaches, and his mother gave him 5 more; how many had he in all? He ate 4; how many had he remaining?

3. Walter had 4 cents, and his father gave him 3 more; how many had he then? He spent 3 cents for 7 plums; how many were left? He ate 5 plums; how many had he left?

LESSON XVI.

1. Recite this Table as both an Addition and Subtraction Table.

UPPER COUNTERS.

LOWER COUNTERS.	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	
3	4	5	6	7		
4	5	6	7			
5	6	7				
6	7					

2. Write the following in Equations, and read them:

5 and 2 are 7 ($5 + 2 = 7$);
4 and 3 are 7;
3 and 4 are 7;
2 and 5 are 7;
7 less 4 equal 3;
7 less 5 equal 2; 7 less 3 equal 4; 7 less 2 equal 5;
6 less 3 equal 3; 5 less 2 equal 3; 5 less 3 equal 2.

Exercises for the Slate and Board.

Addition.

2	2	2	3	1	1	2	5	1	1	2	3
1	3	2	1	5	3	2	1	4	4	4	2
3	2	2	3	1	2	3	1	1	2	1	1

Subtraction.

7	6	7	7	6	6	7	7	6	6	7	5
1	2	5	2	1	4	3	4	3	5	6	2

LESSON XVII.

ADDITION AT SIGHT.

1. If I write *letters*, thus, *o x*, *d o g*, *h o r s e*, you can name the *words*, which they form, at first sight, without stopping to *spell* them.

2. If I write *numbers*, thus, $\begin{array}{c}2\\ \underline{3}\end{array}$, $\begin{array}{c}3\\ \underline{4}\end{array}$, $\begin{array}{c}5\\ \underline{2}\end{array}$, you may become able to name their *Sums*, without stopping to *add*, as readily as you name words.

3. Copy the Exercises on your slate, and name the Sums, going from the left to the right; then from right to left; and finally name them by skipping, in every possible manner. Do not *write* the Sums.

At recitation the Exercises will be written on the blackboard, and you will name the Sums instantly, as your teacher points to the Exercises, one by one.

Addition at Sight.

4	1	5	2	3	3	5	4	6	2	2	3
2	6	1	5	3	4	2	3	1	3	2	2
—	—	—	—	—	—	—	—	—	—	—	—

The *Sums* are the *same* in all cases where the *figures* are the *same*, though the order of the figures be changed. Hence it is necessary only that we *know* WHAT FIGURES *we have*, without regarding the order in which they stand.

Write the missing numbers in the following

EXERCISES FOR THE SLATE AND BOARD.

$5 + ? = 6$	$4 + ? = 6$	$3 + ? = 7$	$6 + ? = 7$
$? - 2 = 5$	$4 - 3 = ?$	$5 - 4 = ?$	$? - 2 = 4$
$2 + ? = 7$	$6 - 5 = ?$	$6 - 3 = ?$	$2 + 2 = ?$
$5 + ? = 7$	$4 + ? = 7$	$? + 2 = 7$	$7 - 4 = ?$
$5 - 2 = ?$	$5 - 3 = ?$	$6 - 2 = ?$	$7 - ? = 2$
$3 + 4 = ?$	$2 + 5 = ?$	$6 - 4 = ?$	$? - 4 = 3$

1. How many more are 7 than 4? Than 2? 6? 1? 3? 5? 4?

2. How many less are 2 than 7? Than 5? 3? 6? 4? 7?

3. How many less are 3 than 5? 4? 7? 6?

4. How many are 3 and 3? 2 and 5? 4 and 2? 3 and 4?

LESSON XVIII.

EQUATIONS.

BY ADDITION.	BY SUBTRACTION.
$2+3=5$, *and* $3+2=5$.	$5-2=3$, *and* $5-3=2$.

1. In the first of the above Equations, 2 + 3 is the FIRST MEMBER, and 5 is the SECOND MEMBER. The two Members of an Equation are always *equal,* and the Sign of Equality stands between them.

2. Since the Sum of 2 and 3 is 5, we may write two Equations by Addition: $2 + 3 = 5$, and $3 + 2 = 5$.

3. Again, since the Sum of 2 and 3 is 5, it is evident that if 2 be subtracted from 5, the Remainder will be 3; and that if 3 be subtracted from 5, the Remainder will be 2. Hence, we write two Equations by Subtraction: $5 - 2 = 3$, and $5 - 3 = 2$.

4. Thus, from the three numbers, **2, 3,** and **5,** we have formed four Equations: $2 + 3 = 5$, $3 + 2 = 5$; $5 - 2 = 3$, and $5 - 3 = 2$.

5. From any three unequal numbers, such that the Sum of the two smaller ones equals the largest, we may form *two* Equations by Addition and *two* Equations by Subtraction.

RULE I.

TO FORM EQUATIONS BY ADDITION:

I.—*For the First Member of an Equation, write the two smaller numbers with the Sign Plus between them.*

II.—*For the Second Member, write the largest of the three numbers, placing the Sign of Equality between the Members.*

III.—*Form the second Equation by Addition from the first, by changing the places of the two smaller numbers.*

Rule II.

To Form Equations by Subtraction:

I.—*For the First Member of an Equation, write the largest of the three numbers, and after it one of the two smaller numbers, placing the Sign Minus between them.*

II.—*For the Second Member, write the other of the two smaller numbers, placing the Sign of Equality between the Members.*

III.—*Form the second Equation by Subtraction from the first, by changing the places of the two smaller numbers.*

Note.—When the two smaller numbers are equal, the two Equations by Addition will be precisely alike, and also those by Subtraction.

In the manner directed by the preceding Rules, form and write four Equations from each of the following

Groups of Three Numbers.

1, 3 and 4;	1, 5 and 6;	1, 6 and 7;	2, 5 and 7;
2, 4 and 6;	1, 4 and 5;	3, 4 and 7;	2, 3 and 5.

If the three numbers are given in an Equation, it is plain that we can form three more Equations from this.

Form three other Equations from each of the following

Equations.

$4 + 2 = 6$;	$5 + 1 = 6$;	$7 - 3 = 4$;	$7 - 1 = 6$;
$7 - 2 = 5$;	$6 - 1 = 5$;	$3 + 2 = 5$;	$4 + 1 = 5$.

Addition at Sight.

2	3	1	2	4	3	3	1	5	2	2	4
2	2	3	3	2	3	4	6	1	5	4	3
—	—	—	—	—	—	—	—	—	—	—	—

LESSON XIX.

1. In this picture, how many peaches are on Willie's table? How many on Mary's table? How many on both? 7 and 1 are how many?

2. How many oranges are on Willie's table? How many on Mary's? How many on both? 6 oranges and 2 oranges are how many?

3. How many pears are on Mary's table? How many on Willie's? How many on both? 5 pears and 3 pears are how many?

4. How many apples are on Mary's table? How many on Willie's? How many on both? 4 apples and 4 apples more are how many?

How many are

1 and 7?	2 and 6?	2 and 3?	5 and 3?	4 and 4?
2 and 1?	3 and 5?	2 and 4?	6 and 2?	3 and 4?

5. If Mary should give Elizabeth her peach, how many peaches would she and Willie have left? 8 peaches less 1 peach are how many?

6. If Willie should give Harry 3 pears, how many would he and Mary have left? 8 pears less 3 pears are how many?

7. If Mary should give Jane 2 oranges, how many would she and Willie have left? 8 oranges less 2 oranges are how many?

8. If Willie should give his 4 apples to his mother, how many would he and Mary have left? 8 apples less 4 apples are how many?

How many are

8 less 2?	8 less 4?	8 less 6?	8 less 3?
8 less 5?	8 less 7?	8 less 1?	7 less 4?

LESSON XX.

Add the numbers in each of the following columns by 5's, in the manner explained on page 27, for adding by 6's:

3	3	4	3	4	4	2	3	1	1	2	3	4
4	5	3	4	3	2	4	1	2	1	2	3	4
2	1	4	5	2	2	1	4	5	1	2	3	4
2	4	1	2	4	4	5	3	4	1	2	3	4
4	2	3	1	2	3	3	4	3	1	2	3	4

Add the following by 6's:

5	3	4	5	2	2	3	5	1	2	3	4	5
4	5	3	4	5	4	1	4	1	2	3	4	5
2	4	2	2	1	2	3	2	1	2	3	4	5
3	5	4	4	3	3	4	4	1	2	3	4	5
5	3	2	5	4	2	3	2	1	2	3	4	5
5	4	3	4	3	5	4	1	1	2	3	4	5

Written Exercises.

1. Walter and Albert went hunting. Walter shot 5 squirrels, and Albert 3; how many did both kill? They lost 2 of the squirrels; how many were left? Walter killed 6 pigeons, and Albert 2; how many did both kill? They gave away 4 of the pigeons; how many were left?

2. Anna, Amelia, and Willie went to gather flowers. Anna picked 4 lilies, and Willie gave her 4 more; how many had she in all? She lost 3; how many had she left? Amelia picked 7 lilies, and Willie gave her 1; how many had she then? She gave her mother 5 lilies; how many had she at last?

LESSON XXI.

Write the following in Equations, and read them:

7 and 1 are 8,	3 and 5 are 8,	7 less 6 equal 1;
6 and 2 are 8,	2 and 6 are 8,	8 less 5 equal 3,
5 and 3 are 8,	1 and 7 are 8;	8 less 7 equal 1,
4 and 4 are 8,	7 less 3 equal 4,	8 less 4 equal 4,
4 and 3 are 7;	7 less 5 equal 2,	8 less 6 equal 2,
6 and 1 are 7,	7 less 1 equal 6,	8 less 3 equal 5,
2 and 5 are 7,	7 less 2 equal 5,	8 less 1 equal 7,
3 and 4 are 7,	7 less 4 equal 3,	8 less 2 equal 6.

Write four Equations from each of the following

Groups of Three Numbers.

1, 7, and 8; 2, 6, and 8; 3, 5, and 8; 2, 5, and 7 3, 4, and 7.

Write three others from each of the following

Equations.

$2 + 6 = 8$; $4 + 3 = 7$; $5 + 3 = 8$; $8 - 5 = 3$;
$1 + 7 = 8$; $8 - 2 = 6$; $7 - 3 = 4$; $2 + 5 = 7$.

Exercises for the Slate and Board.

Addition.

I.

1	2	2	3	2	5	2	2	1	5	1	6	4
3	3	4	3	2	2	1	5	5	1	6	1	2
4	2	2	2	4	1	5	1	2	2	1	1	2

II.

3	2	3	4	4	4	2	3	3	3	4	1	2
1	3	2	3	1	2	4	3	2	1	1	4	1
4	3	3	1	3	1	1	1	2	3	2	3	4

Subtraction.

6	5	6	6	7	8	8	7	8	8	7	8	7
4	2	3	2	2	3	5	3	2	4	4	6	5

LESSON XXII.

EQUATIONS.

$2 + ? = 8$	$5 + ? = 7$	$7 - 2 = ?$	$4 + ? = 8$
$? + 3 = 8$	$8 - 2 = ?$	$7 - ? = 4$	$3 + ? = 8$
$3 + ? = 8$	$8 - ? = 5$	$? - 4 = 3$	$? + 4 = 8$
$2 + 5 = ?$	$? - 4 = 4$	$7 - 3 = ?$	$7 - 4 = ?$
$3 + ? = 7$	$8 - 5 = ?$	$7 - 5 = ?$	$? - 3 = 5$
$? + 4 = 7$	$? + 6 = 8$	$6 + 2 = ?$	$? + 2 = 8$

Add the numbers in the following columns by 7's, in the manner explained on page 27, for adding by 6's:

4	5	6	5	4	2	5	4	5	3	5	4	5
5	5	3	5	5	4	1	6	6	2	6	6	4
4	3	4	2	4	4	6	6	6	6	4	3	3
3	4	5	3	6	5	4	6	6	5	5	6	5
5	4	3	6	2	6	5	6	5	5	1	2	4

LESSON XXIII.

1. Learn this Addition and Subtraction Table.

	1	2	3	4	5	6	7
1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	
3	4	5	6	7	8		
4	5	6	7	8			
5	6	7	8				
6	7	8					
7	8						

2. If we write the Equation $3 + 4 = 7$ in the form of an Example in Addition, as shown at the right, the order of the numbers is not changed. Hence we may use the method shown on page 23 also *To find any Number in an Example in Addition, when missing;* thus:

$$\left.\begin{array}{r} 3 \\ \underline{4} \end{array}\right\} \textit{Parts.} \qquad \begin{array}{r} 7 \end{array} \ \textit{Sum.}$$

Missing Numbers.	Names.	Methods of Finding.
FIRST.	PARTS.	*Subtract* the SECOND from the THIRD.
SECOND.	PARTS.	*Subtract* the FIRST from the THIRD.
THIRD.	SUM.	*Add* the FIRST and SECOND.

EXERCISES FOR THE SLATE AND BOARD.

I.

2	5	3	5	3	?	?	?	?	4	?	?	6
3	3	?	?	?	2	5	4	2	?	3	3	?
?	?	7	8	8	7	8	7	8	8	7	8	8

II.

3	?	3	?	2	2	?	?	5	4	3	?	?
?	5	?	4	?	?	2	6	?	?	?	1	1
5	7	6	8	6	8	6	7	6	6	4	8	4

LESSON XXIV.

If we take the Equation $8 - 3 = 5$, and write it in the form of an Example in Subtraction, as shown at the right, the order of the numbers is not changed. Hence we may use the method shown on page 24 also *To find a missing number in an Example in Subtraction;* thus:

8 *Minuend.*
3 *Subtrahend.*
5 *Difference.*

Missing Numbers.	Names.	Methods of Finding.
FIRST.	MINUEND.	*Add* the SECOND and THIRD.
SECOND.	SUBTRAHEND.	*Subtract* the THIRD from the FIRST.
THIRD.	DIFFERENCE.	*Subtract* the SECOND from the FIRST.

EXERCISES FOR THE SLATE AND BOARD.

I.

7	7	8	8	8	?	?	?	?	?	?	?
3	4	3	5	4	5	2	4	3	6	3	2
?	?	?	?	?	2	5	3	3	2	5	6

II.

7	8	6	7	?	7	7	8	?	?	8	6
?	?	3	?	5	?	?	?	3	4	?	?
2	3	?	4	3	5	3	4	4	4	5	3

In the same manner as we added by 7's on page 37, add each column by 8's in the following

EXERCISES FOR THE SLATE AND BOARD.

7	7	6	5	4	6	7	6	6	5	4	5
5	3	6	5	7	6	5	5	6	5	3	4
2	2	3	4	4	3	2	4	3	3	6	5
5	6	7	3	6	5	3	5	4	7	4	7
5	6	2	7	3	4	7	4	5	4	7	3

LESSON XXV.

1. In this picture, how many beehives in each of the two rows? How many in both rows? You see one hive standing apart from the two rows. If you count this with the others, how many are there? We make a ***figure Nine*** thus: ***9.*** Make one. One and how many more make 9?

2. How many birds do you see over this house? How many are about to light on the tree? How many birds in all? 7 birds and 2 more are how many?

3. How many squirrels have climbed the tree? How many others are running towards the tree? 4 squirrels and 2 more are how many? How many squirrels are on the fence, running away from the tree? How many squirrels in all?

4. How many windows in the upper story, in the side of the house? How many in the lower story, in the side of the house? How many windows are 3 and 2 more? How many windows do you see in the and of the house? 5 and 4 more are how many? If a door should be put in place of one of the windows, how many would be left? 1 window from 9 leaves how many?

5. If the boy shoot two of the squirrels, how many will be left? 2 from 9 leave how many?

6. If 3 hives be carried away, how many will remain? 3 from 9 leave how many?

7. If the boy shoot 4 of the birds, how many will be left? 4 from 9 leave how many?

Written Exercises.

1. Harry and Edward went fishing. Harry caught 2 sunfish, and Edward 7; how many did both catch? 2 and 7 more are how many? 3 of the fishes were lost from the basket; how many were left? 3 from 9 leave how many?

2. Harry caught 3 perch, and Edward 6; how many did both boys catch? 3 and 6 are how many? They gave away 5 perch; how many were left? 5 perch from 9 leave how many?

3. Harry caught 5 bass, and Edward 4; how many were caught in all? 5 and 4 are how many? They gave away 2 bass; how many were left? 2 from 9 leave how many? They also lost 2 bass. 4 bass from 9 leave how many?

4. Harry caught 6 eels, and lost 4 of them; how many had he left? He then caught 2 more; how many had he at last? Edward caught 5 eels; how many eels had Harry and Edward, to carry home? 4 eels and 5 eels are how many?

LESSON XXVI.

Exercises for the Slate and Board.

Addition.

I.

6	4	5	2	3	2	3	5	5	3	7	2	6	2
2	3	3	4	4	1	4	2	2	3	1	2	1	4
1	2	1	2	1	6	2	1	2	3	1	5	1	3

II.

3	2	6	2	5	4	3	3	2	3	3	4	3
2	3	1	2	1	1	1	2	4	2	1	1	1
3	1	1	2	2	2	2	1	2	1	2	3	4
1	2	1	2	1	1	2	3	1	2	1	1	1

Subtraction.

9	7	8	7	9	8	7	9	8	7	9	8	9	9
3	2	3	3	2	4	4	5	2	5	6	5	7	4

Equations.

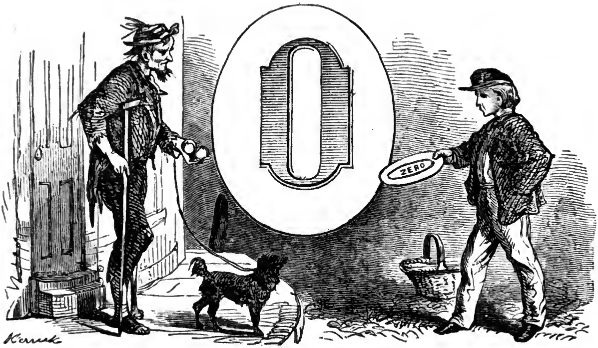
7 + ? = 9	? + 2 = 9	9 − 3 = ?	9 − 2 = ?
9 − 5 = ?	? − 3 = 6	9 − ? = 5	9 − ? = 3
4 + ? = 9	9 − ? = 2	? − 5 = 4	? − 4 = 5
5 + ? = 9	3 + ? = 9	9 − ? = 4	9 − ? = 6

Examples in Addition.

3	7	3	5	?	?	4	2	1	2	?	2	?	?	4	?	?
6	?	?	?	2	4	?	7	?	?	3	?	5	4	?	7	2
9	9	8	9	9	9	8	?	9	8	9	9	9	8	9	9	8

Examples in Subtraction.

9	9	9	8	9	9	8	?	?	9	?	9	?	?	9	?	8
6	2	?	?	4	?	?	7	8	?	6	?	5	4	?	2	?
3	?	7	3	?	5	4	2	1	3	3	2	4	4	4	7	2



LESSON XXVII.

1. Frank had 2 apples on a fruit-dish. He took 2 apples from the dish and gave them to a beggar. How many apples were left on the dish? 2 apples from 2 apples leave how many apples? We use a figure written thus, ***0,*** to stand for ***Nothing.*** Its name is ***Zero,*** or ***Naught,*** or ***Cipher.*** Each of these words means NOTHING; and the figure, ***0,*** also means *Nothing.*

2. In Arithmetic we use no figures but these: ***0, 1, 2, 3, 4, 5, 6, 7, 8, 9.***

Write the following in Equations; and from each Equation so written write three others in the manner explained in Lesson XVIII.

5 and 4 are 9,	7 and 2 are 9,	4 and 5 are 9,
6 and 3 are 9;	2 and 7 are 9;	3 and 6 are 9;
9 less 3 equal 6,	9 less 5 equal 4,	9 less 2 equal 7,
9 less 6 equal 3;	9 less 4 equal 5;	9 less 7 equal 2.

Learn and recite the annexed Addition and Subtraction Table:

Add the numbers in the following columns by 9's:

6	3	7	6	4	4
5	6	8	6	5	2
7	7	2	4	6	5
3	5	5	8	6	7
5	4	5	5	5	3
3	6	6	3	8	7
7	5	3	4	2	8

	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	
3	4	5	6	7	8	9		
4	5	6	7	8	9			
5	6	7	8	9				
6	7	8	9					
7	8	9						
8	9							

LESSON XXVIII.

ADDITION.

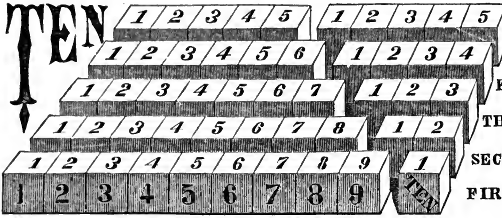
1. In the cut on the next page, in the first row, how many cubes in the greater part? How many in the other part? How many, in all, in that row? 9 cubes and 1 more are how many?

2. In the second row, how many cubes in the greater part? How many in the other part? How many, in all, in the second row? 8 cubes and 2 more are how many? 2 and 8 more?

3. In the third row, how many cubes in the greater part? In the other part? In the third row? 7 cubes and 3 more are how many? 3 and 7 more?

4. In the fourth row, how many cubes in the greater part? In the other? In the fourth row? 6 cubes and 4 more are how many? 4 and 6 more?

5. In the fifth row, how many cubes in each part? 5 cubes and 5 more are how many?



SUBTRACTION.

6. If we take 1 cube from the first row, how many will be left? If, instead, we take away 9 cubes, how many will remain? 1 cube from ***Ten*** cubes leaves how many? 9 cubes from TEN cubes leave how many?

7. How many cubes will be left in the second row, if we take away 2 cubes? If we take 8? 2 cubes from TEN leave how many? 8 from TEN how many?

8. How many cubes will be left in the third row, if we take away 3? If we take 7? 3 cubes from TEN leave how many? 7 from TEN how many?

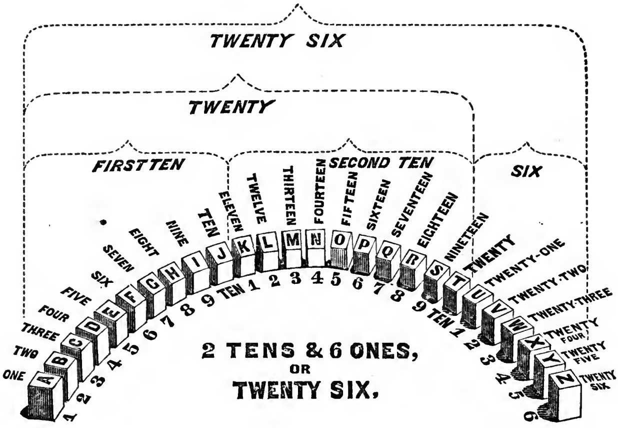
9. Four cubes taken from the fourth row will leave how many? 6 taken away leave how many? 4 cubes from TEN leave how many? 6 from TEN how many?

10. Five cubes taken from the fifth row will leave how many? 5 cubes taken from TEN leave how many?

Learn the following

TABLE:

9 and 1 are TEN,	1 from TEN leaves 9,
8 and 2 are TEN,	2 from TEN leave 8,
7 and 3 are TEN,	3 from TEN leave 7,
6 and 4 are TEN,	4 from TEN leave 6,
5 and 5 are TEN,	5 from TEN leave 5,
4 and 6 are TEN,	6 from TEN leave 4,
3 and 7 are TEN,	7 from TEN leave 3,
2 and 8 are TEN,	8 from TEN leave 2,
1 and 9 are TEN;	9 from TEN leave 1.



LESSON XXIX.

1. Walter arranged his set of Alphabet Blocks on a sheet of paper, in the order shown above. He then proceeded to count them, printing the numbers on the paper in *words,* just above the blocks. You see that there were ***Twenty-six*** blocks.

2. He counted them again, and wrote the numbers just below the blocks, in *figures.* For the blocks A, B, C, D, E, F, G, H, I, he counted and wrote 1, 2, 3, 4, 5, 6, 7, 8, 9. For the block J he counted TEN, as before; but, not finding any single *figure* for TEN, he printed the *word* TEN. Counting the blocks K, L, M, N, O, P, Q, R, S, T, he wrote below them 1, 2, 3, 4, 5, 6, 7, 8, 9, TEN.

For the blocks U, V, W, X, Y, Z, he counted and wrote 1, 2, 3, 4, 5, 6.

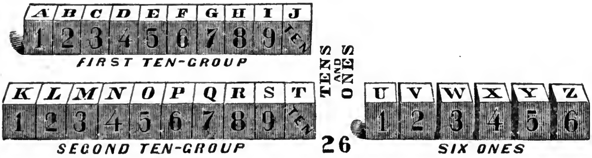
Finding that the two sets of numbers were unlike beyond TEN, and not knowing how to write a number

greater than Nine, in *figures*, he was at first puzzled. Finally, he wrote and gave to his teacher the following

TABLE.

TEN	2 TENS
	are Twenty,
and 1 are Eleven,	and 1 are Twenty-one,
and 2 are Twelve,	and 2 are Twenty-two,
and 3 are Thirteen,	and 3 are Twenty-three,
and 4 are Fourteen,	and 4 are Twenty-four,
and 5 are Fifteen,	and 5 are Twenty-five,
and 6 are Sixteen,	and 6 are Twenty-six.
and 7 are Seventeen,	
and 8 are Eighteen,	
and 9 are Nineteen.	

LESSON XXX.



TWENTY-SIX.

Mary counted and numbered TEN of her Alphabet Blocks, and, naming them a ***Ten-group,*** placed them at her left hand. Forming a second *Ten-group,* she placed it with the first. The remaining *Six* blocks she numbered and placed at the right.

Said she to her teacher: "I have *Six* single blocks, standing at the right. I will write a figure 6, to stand for these. Since I have *Two* Ten-groups, I will write a figure 2, to stand for them; remembering that this 2

does not stand for 2 *blocks,* but for 2 *Ten-groups,* each having Ten blocks.

"Since the 2 *Ten-groups* are placed at the *left* of the 6 *single blocks,* I will write the *figure 2,* which stands for the 2 Ten-groups, at the *left* of the *figure 6,* which stands for the 6 single blocks; thus, ***26.***"

"That is correct," answered the teacher. "Two TENS are named TWENTY. Your figures, 26, are to be read *Twenty-six.* For any number of Ones, or single blocks which are not more than 9, you write one figure. Then, if you have any number of Tens which are not more than 9, you write one figure for them; placing it at the *left* of the figure which stands for the *Ones.* If there are *no* Ones, you write a *Cipher,* thus, 0, at the right of the figure standing for the Tens. 78 equals 7 Tens and 8 Ones; 90 equals 9 Tens and *no* (0) Ones."

Some numbers are named as shown in the following

TABLE:

3 Tens		Thirty,	3 Tens and 1 One		Thirty-one, etc.
4 Tens	*are named*	Forty,	4 Tens and 1 One	*are named*	Forty-one, etc.
5 Tens		Fifty,	5 Tens and 1 One		Fifty-one, etc.
6 Tens		Sixty,	6 Tens and 1 One		Sixty-one, etc.
7 Tens		Seventy,	7 Tens and 1 One		Seventy-one, etc.
8 Tens		Eighty,	8 Tens and 1 One		Eighty-one, etc.
9 Tens		Ninety.	9 Tens and 1 One		Ninety-one, etc.

Some numbers greater than 9 are written thus:

TABLE.

By Words.	By Figures.	By Words.	By Figures.	By Words.	By Figures.
Ten,	10;	Fourteen,	14;	Eighteen,	18;
Eleven,	11;	Fifteen,	15;	Nineteen,	19;
Twelve,	12;	Sixteen,	16;	Twenty,	20;
Thirteen,	13;	Seventeen,	17;	Twenty-one,	21;

By Words.	By Figures.	By Words.	By Figures.	By Words.	By Figures.
Twenty-two,	22;	Thirty-one,	31;	Seventy,	70;
Twenty-three,	23;	Forty,	40;	Eighty,	80;
Twenty-four,	24;	Fifty,	50;	Ninety,	90;
Thirty,	30;	Sixty,	60;	Ninety-nine,	99.

LESSON XXXI.

Write the following Numbers in Figures:

Twenty-seven;	Fifty-three;	Forty;
Thirty-three;	Eleven;	Thirty-four;
Fifteen;	Sixty-eight;	Seventeen;
Thirty-nine;	Twenty;	Nineteen;
Fourteen;	Seventy-seven;	Ninety-three;
Forty-five;	Thirty;	Fifty;
Thirteen;	Eighty-six;	Ninety-nine.

Read aloud the following Numbers:

17	41	50	64	55	36	69	58	60
27	44	13	12	91	10	9	84	80
37	49	72	85	11	93	47	39	99

Exercises for the Slate and Board.

Addition.

3	4	2	5	2	2	3	2	4	3	4	6	4
4	2	3	1	2	1	5	4	3	1	3	1	5
3	3	5	4	5	7	2	4	3	6	2	3	1

Subtraction.

9	10	8	10	10	10	9	10	10	9	9
2	2	4	5	3	6	5	4	7	3	4

Equations.

$5+5=?$	$5+?=10$	$10-5=?$	$?-5=5$
$?+4=10$	$6+?=10$	$10-?=4$	$?-3=1.$
$10-?=3$	$?-7=3$	$?+2=10$	$?-2=8$

LESSON XXXII.

1. Learn and recite the annexed Table:

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	
4	5	6	7	8	9	10	11		
5	6	7	8	9	10	11			
6	7	8	9	10	11				
7	8	9	10	11					
8	9	10	11						
9	10	11							

2. 2 *Ones* and 3 *Ones* are 5 *Ones;* and 2 *Tens* and 3 *Tens* are 5 *Tens,* or 50. 3 *Ones* taken from 7 *Ones* leave 4 *Ones;* and 3 *Tens* taken from 7 *Tens* leave 4 *Tens,* or 40.

In *every* case, *Tens* are added to *Tens,* or subtracted from *Tens,* in the same manner as *Ones* are added to or subtracted from *Ones.*

Exercises for the Slate and Board.

Addition.

I.

11	21	32	43	49	54	37	65	46	81	76
12	13	23	34	30	22	52	31	23	17	12

II.

21	15	30	51	11	33	12	42	52	16	71
14	32	24	16	23	22	23	13	24	62	14
32	40	15	21	45	11	34	24	12	20	14

Subtraction.

22	33	45	52	58	65	87	76	69	88	99
11	22	33	20	33	41	32	45	35	73	33

LESSON XXXIII.

Mental Exercises.

$9 + 2 = ?$	$? + 4 = 11$	$8 + 3 = ?$	$? + 5 = 11$
$8 + ? = 11$	$6 + ? = 11$	$2 + ? = 11$	$7 + ? = 11$
$6 + 5 = ?$	$? + 8 = 11$	$? + 7 = 11$	$? + 6 = 11$
$3 + ? = 11$	$5 + ? = 11$	$? + 2 = 11$	$4 + ? = 11$
$? + 3 = 11$	$11 - 2 = ?$	$? + 9 = 11$	$7 + 4 = ?$
$11 - 4 = ?$	$11 - 6 = ?$	$11 - 3 = ?$	$11 - 5 = ?$
$11 - 8 = ?$	$11 - 1 = ?$	$11 - 7 = ?$	$11 - 9 = ?$
$10 - 3 = ?$	$10 - 5 = ?$	$10 - 6 = ?$	$10 - 4 = ?$

Exercises for the Slate and Board.

1. Frank had 4 rabbits, and his father gave him 7 more; how many had he in all?

2. A farmer had 16 sheep in one pasture, and 22 in another; how many had he in both? He sold 25 sheep; how many had he left?

3. In a school there were 43 boys, and 35 girls; how many pupils in all? 15 pupils left; how many remained?

4. Willie found a hen's nest with 15 eggs, and Walter found one with 13; how many eggs did both find?

5. In one flock of pigeons were 54, and in another 35; how many pigeons in both flocks?

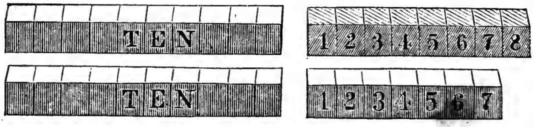
Addition.

52	34	18	26	57	84	45	74	56	64
23	15	21	41	31	13	23	22	23	14

Subtraction.

74	98	76	69	48	87	36	66	52	75
31	35	43	25	15	36	14	33	40	25

LESSON XXXIV.



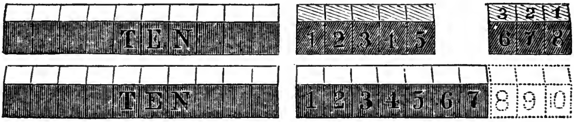
1. In the upper row of these cubes, how many cubes are in the greater part? How many in the other part? In both parts?

2. In the lower row, how many cubes are in the greater part? In the other part? In both?

3. We wish to find how many cubes there are in all; that is, find the Sum of 18 and 17.

1st. We first add the 7 cubes in the lower row, and the 8 in the upper row. Since 7 and 3 are 10, we take 3 of the 8 cubes, and, adding them to the 7, have 10 cubes. Since 3 taken from 8 leave 5, we have 5 of the 8 cubes left.

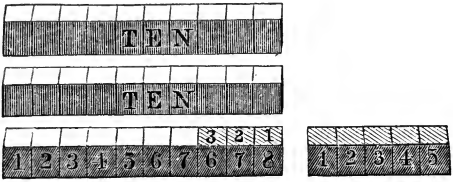
As shown in the following cut,



we separate the 8 cubes into 3 cubes and 5 cubes, and then place the 3 cubes at the end of the row of 7 cubes. Counting them together, we have 10 cubes.

Thus we have in all a group of 10 cubes, and 5 single cubes; which are 15 cubes.

2d. We next arrange the cubes as shown in the cut at the top of the opposite page. The Ten-group, which we have formed, we CARRY TO THE LEFT and place with



the 2 other Ten-groups. Counting the 2 Tens with the 1 Ten CARRIED, we find they are 3 Tens. Thus, we have, in all, 3 Tens and 5 Ones; which are 35 cubes.

In like manner, we add 18 and 17 by figures. We write the numbers as shown at the right. Taking 3 of the 8 Ones, we add them to the 7 Ones to make 10, and then add the remaining 5 Ones to the 10, and thus obtain 15 as the Sum of 8 and 7. We then CARRY TO THE LEFT the 1 Ten of the 15, and, adding it with the 1 Ten of the 17 and the 1 Ten of the 18, obtain 3 Tens. Thus we find that the Sum of 18 and 17 is 3 Tens and 5 Ones, or 35.

ADDITION.

$$\left.\begin{array}{r} 18 \\ \underline{17} \end{array}\right\} \textit{Parts.}$$
$$35 \quad \textit{Sum.}$$

In like manner we add any *two or more numbers*, when their Sum is not greater than 99. We CARRY TO THE LEFT all the *Tens* formed by adding the figures in the right-hand column.

TESTING THE SUM.

After adding numbers we sometimes doubt the correctness of our work. In such cases it is well to add the figures a second time, commencing at the top and adding to the bottom. If we obtain the same result as in the first instance, it is presumed that we have found the true Sum. This second addition is named ***Testing the Sum.***

LESSON XXXV.

This Table should be learned so thoroughly that any part of it can be given without hesitation. Give special attention to that part of the Table which includes numbers greater than 10. No further Table for Addition or Subtraction will be needed if this be mastered.

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Exercises for the Slate and Board.

Addition.

I.

4	25	16	37	48	56	27	57	46	54
6	65	34	13	12	35	16	26	37	29

II.

8	23	39	63	18	19	24	11	62	42
7	50	23	14	25	44	18	45	18	19
5	17	28	19	34	23	35	37	16	27

III.

9	14	16	12	34	12	11	14	13	19
6	35	13	14	13	14	14	31	27	33
8	13	47	17	28	36	42	14	18	22
7	28	14	33	13	27	19	28	30	11

LESSON XXXVI.

Add by 7's.			Add by 8's.		Add by 9's.		Add by 10's.		Add.	
6	3	1	7	6	7	8	8	7	15	18
5	6	6	5	7	8	7	9	8	24	14
4	2	5	6	5	6	5	5	6	10	23
5	4	4	2	7	2	6	6	5	27	17
1	6	5	4	7	4	1	2	4	14	25

WRITTEN EXERCISES.

1. How many bushels of apples in 3 piles, the first containing 26 bushels, the second 35 bushels, and the third 29 bushels?

2. A poultry dealer sold some turkeys for 46 dollars, some geese for 23 dollars, and some chickens for 27 dollars; how many dollars did he receive in all?

3. A farmer sold 38 bushels of wheat, 26 bushels of corn, 17 bushels of rye, and 16 bushels of barley; how many bushels of grain did he sell in all?

4. In 3 days, Albert picked 23 quarts of strawberries, Harry 29 quarts, Alfred 34 quarts, and Walter 10 quarts; how many quarts did the 4 boys pick?

5. How many pigeons are twenty-eight pigeons, thirty-four pigeons, and twenty-nine pigeons?

6. Frank rode twenty-nine miles Monday, thirty Tuesday, and eighteen Wednesday; how many miles did he ride during the three days?

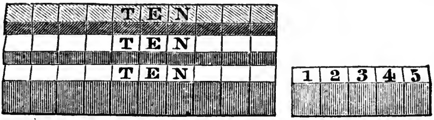
7. Willie had 47 peaches in his basket, Frederick 16 in his, and Henry 28 in his; how many peaches had the three boys?

8. Homer had 45 oranges in his basket, and Horatio 39 in his; how many oranges had both boys?

9. Nellie spelled 39 words, and Mary 48; how many did both spell?

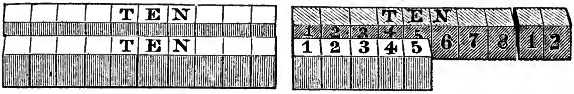
LESSON XXXVII.

I.—MINUEND.

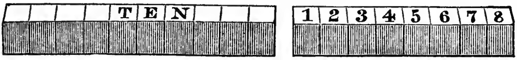


SUBTRAHEND.

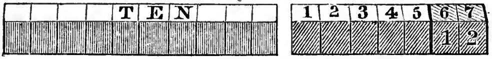
II.—MINUEND (REARRANGED).



SUBTRAHEND.



DIFFERENCE.



SUBTRACTION.

The right-hand figure in the Subtrahend greater than the figure above it in the Minuend.

EXAMPLE. From 35 subtract 18. Writing the 18 under the 35, we find that 8 cannot be subtracted from 5.

SUBTRACTION.

35 *Minuend.*
18 *Subtrahend.*

First: Subtraction by Objects.

The above cut is in 2 Parts. In Part I. how many Ten-groups are in the Minuend? How many single cubes? 3 Tens and 5 Ones are how many? How many Ten-groups are in the Subtrahend? How many single cubes? 1 Ten and 8 Ones are how many?

This Minuend, then, having 35 cubes, and Subtrahend having 18 cubes, answer to the Minuend and Subtrahend in the given Example. Hence the Difference between 35 cubes and 18 cubes will be the answer to the given Example.

Since 8 cubes cannot be subtracted from 5 cubes, we take 1 of the 3 Ten-groups in the Minuend and CARRY it TO THE RIGHT, and place it with the 5 cubes, as 10 separate cubes.

This Minuend, as re-arranged, is shown in Part II. of the cut. Since the Minuend and Subtrahend in Part II. are equal to those in Part I., the Difference in Part II. and in Part I. must be the same. We will find the Difference in Part II.

We separate the Ten-group, which stands with the 5 cubes, into two parts, one having 8 cubes and the other 2. Since there are 8 cubes in the Subtrahend, we subtract 8 cubes from the 10 cubes. There are then left, in the Minuend, 2 of the 10 cubes and also the 5 cubes. Placing 5 cubes and 2 cubes in the Difference, we have 7 cubes as the Difference between the 8 cubes of the Subtrahend and the 10 cubes and 5 cubes of the Minuend.

In Part I. we had 1 Ten to subtract from 3 Tens; but in Part II., having CARRIED TO THE RIGHT 1 Ten in the Minuend, we have only 2 Tens remaining at the left. Subtracting 1 Ten from 2 Tens, and obtaining 1 Ten for the Difference, we place 1 Ten-group in the Difference. Thus we find that 18 cubes taken from 35 cubes leave 17 cubes.

Second : Subtraction by Figures.

What we have done with the cubes we will now do with the figures. As shown in the margin, we write

the 18 under the 35. We then re-arrange the Minuend, by first taking 1 of the 3 Tens and CARRYING it TO THE RIGHT and writing it as 10 Ones, over the 5 Ones, and then drawing a line through the 3, to show that it is not to be used, and writing the 2 remaining Tens over the former 3.

SUBTRACTION.

$\overset{2}{\cancel{3}}$	$\overset{10}{5}$	*Minuend.*
1	8	*Subtrahend.*
1	7	*Difference.*

First, we subtract 8 from 5 and 10 taken together. Subtracting, 8 from 10 leave 2; and this 2 and the 5, added together, give 7 Ones for the Difference. Subtracting 1 Ten from 2 Tens, we have 1 Ten for the Difference in Tens. Thus we obtain for our Difference 1 Ten and 7 Ones; or 17.

In every Example like this, the work is performed in the manner just explained. It is not necessary, however, to change the figures of the Minuend. In this Example, we write the numbers as shown in the margin. Subtracting, we say, *not* 8 from 5, but 8 from 10 leave 2; 2 and 5 are 7; and then write 7 in the Difference. Finally, we say, *not* 1 Ten from 3 Tens, but 1 Ten from 2 Tens leaves 1 Ten; and write 1 Ten in the Difference.

SUBTRACTION.

35	*Minuend.*
18	*Subtrahend.*
17	*Difference.*

EXAMPLES FOR THE SLATE AND BOARD.

I.

29	54	35	72	91	57	74	32	96
13	29	18	36	43	29	68	16	48

II.

65	82	23	44	63	56	71	62	78
43	37	15	28	27	28	58	33	39

LESSON XXXVIII.

SUBTRACTION.

Testing the Difference.

If the Difference between two numbers be added to the less, the Sum will equal the greater. If this Difference be subtracted from the greater number, the result will equal the less. Hence, we may *Test our Difference* in Subtraction by either of two methods.

First Method: Testing by Addition.

Add the Difference to the Subtrahend. If the Sum equals the Minuend, it is presumed that we have found the true Difference.

Second Method: Testing by Subtraction.

Subtract the Difference from the Minuend. If the result equals the first Subtrahend it is presumed that the first Difference is correct.

Find and test the Difference in each of the following

Examples for the Slate and Board.

Testing by First Method.

82	52	70	93	80	47	90	75	65
64	38	36	79	39	18	45	25	47

Testing by Second Method.

33	79	43	53	39	62	71	80	91
19	49	28	14	19	53	52	65	74

Addition at Sight.

5	7	9	6	4	8	4	5	9	3	5	5	6
4	4	4	4	4	4	5	5	5	5	6	8	3

LESSON XXXIX.

Addition.

34	19	24	18	37	19	23	13	12
15	23	17	32	14	25	11	41	29
16	16	15	13	13	13	23	13	29
17	28	17	29	28	17	35	32	29

Subtraction.

82	94	64	77	85	97	54	78	91
57	31	24	59	25	88	27	59	42

Written Exercises.

Addition and Subtraction.

1. Edwin had a set of 26 Alphabet Blocks on his desk, and Susan had the same number on her desk; how many blocks did both have?

Edwin put 17 of his blocks in the box; how many were left on his desk?

Susan put 19 of her blocks in the box; how many were left on her desk?

How many did both put in their boxes?

How many remained on both desks?

2. Mr. Newton had 55 sheep, and Mr. Lawton had 42; how many sheep did both have?

How many more sheep had Mr. Newton than Mr. Lawton?

Mr. Newton sold 29 sheep to Mr. Lawton; how many had Mr. Lawton then?

How many had Mr. Newton left?

How many more had Mr. Lawton than Mr. Newton?

Addition at Sight.

7	9	7	8	6	5	9	7	4	8	6	6	6
5	6	6	6	6	7	7	7	7	7	9	7	8

LESSON XL.

EXAMPLES FOR THE SLATE AND BOARD.

Addition.

18	27	13	16	15	16	17	18	19
13	12	28	22	15	16	17	18	19
17	14	12	11	15	16	17	18	19
18	25	17	35	15	16	17	18	19
19	16	25	13	15	16	17	18	19

MENTAL EXERCISES.

1. One of 2 hens has 7 chickens, and the other 9; how many chickens have both hens?

2. Charles has 9 marbles, and Frank 6; how many marbles have both boys?

3. Flora picked 8 quarts of strawberries, and Ella 9; how many quarts did both pick?

4. Edward shot 11 squirrels, and Henry 6; how many more did Edward shoot than Henry?

How many did both boys shoot?

5. Harry bought 12 peaches, and gave 5 of them to his sister; how many had he left?

6. Walter was 18 years old, and Willie 9; how many years was Walter older than Willie?

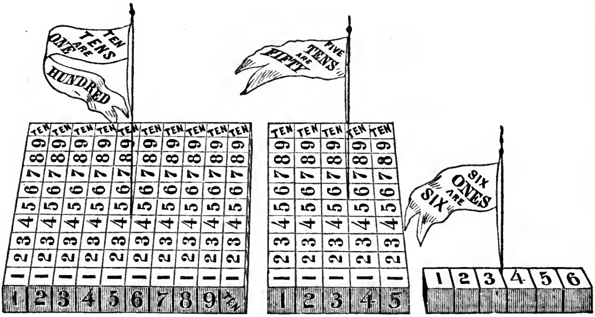
7. On a tree were 17 pigeons, and 9 of them flew away; how many were left on the tree?

Addition at Sight.

2	7	3	5	8	7	4	8	5	8	9	8	3
8	4	8	7	5	7	8	6	8	7	5	8	9

Subtraction at Sight.

4	5	5	6	6	6	7	7	8	7	7	8	8
2	3	2	3	2	4	2	4	2	3	5	4	6



ONE HUNDRED and FIFTY- SIX;
or, in figures,

156.

LESSON XLI.

A maker of Alphabet Blocks prepared 6 sets for lettering, and requested his little daughter to count them and tell him how many there were. She counted them, wrote on them, and arranged them as shown in the above cut.

Said she to her father: "Because I cannot count more than Ten, I have counted the blocks in groups, each having Ten blocks. I have placed each Ten in a row, writing the numbers on the blocks as I counted them, and finally arranged the groups side by side. After making as many Tens as I could, I placed the remaining blocks at the right of these, and counted them 1, 2, 3, 4, 5, 6.

"I have tried to count my Ten-groups, but find there are more than Ten of them. When, at first, I was counting the *single blocks,* I counted them in groups of *Ten single blocks,* because I could not count more than Ten. Now, also, when I am counting my *Ten-groups,* because I cannot count beyond Ten, I will, in the same

manner, count my *Ten-groups* into larger groups, each having *Ten Ten-groups*. As I count these Ten-groups I number them, and write on the ends of them 1, 2, 3, 4, 5, 6, 7, 8, 9, Ten. These *Ten* Ten-groups I separate from the others, and remove them a little to the left, and call them 1 large group. I count the other Ten-groups, and write on them 1, 2, 3, 4, 5.

"Since there are *six* single blocks *at the right,* I will write a *figure 6,* to stand for them; thus, 6. Since there are *five* Ten-groups standing by themselves, *at the left* of the six blocks, I will write a *figure 5* to stand for them; *placing it at the left of the figure 6.* Since there is *one* large group, *at the left of the 5 Ten-groups,* I will write a *figure 1* to stand for this; placing it *at the left of the figure 5.*

"My figures will then be *placed in the same order* as the groups and single blocks; thus, 1 5 6."

Her father said: "Your *large* group is named ***One Hundred;*** your 5 Ten-groups are named Fifty; and your figures, 156, are read *One Hundred and Fifty-six.*"

Numbers greater than 99 are named and written thus:

TABLE.

10 Tens are One	Hundred;	written, 100.
20 Tens are Two	Hundred;	written, 200.
30 Tens are Three	Hundred;	written, 300.
40 Tens are Four	Hundred;	written, 400.
50 Tens are Five	Hundred;	written, 500.
60 Tens are Six	Hundred;	written, 600.
70 Tens are Seven	Hundred;	written, 700.
80 Tens are Eight	Hundred;	written, 800.
90 Tens are Nine	Hundred;	written, 900.

101 is read One Hundred and One.
570 is read Five Hundred and Seventy.
999 is read Nine Hundred and Ninety-Nine.

LESSON XLII.

Write, in figures, the following:

Five Hundred and Twenty-three; Four Hundred and Eighty-seven; Six Hundred and Ninety; Eight Hundred and Forty; Seven Hundred and Ten; Nine Hundred and Ninety-nine; Seven Hundred and Eleven; Five Hundred and Seven; Seven Hundred; Three Hundred and Eight;

Addition.

EXAMPLE. Find the Sum of 456 and 231.

EXPLANATION. We add the Ones and Tens in the same manner as if there were no Hundreds, and finding the Sum to be 87 write it under the columns. Adding 2 Hundreds and 4 Hundreds in the same manner as we added Ones, and Tens, and writing the Sum, 6 Hundreds, we have 687 as the entire Sum of 456 and 231.

ADDITION.

$$\left.\begin{array}{r}456\\ \underline{231}\end{array}\right\}\ \textit{Parts.}$$
$$687\ \ \textit{Sum.}$$

EXERCISES FOR THE SLATE AND BOARD.

I.

147	238	546	329	176	415	824	213
236	354	234	453	215	127	159	158

II.

215	423	154	231	119	235	310	407
326	135	218	154	218	144	207	200
144	218	323	208	351	216	156	126

Subtraction.

We subtract Hundreds from Hundreds in the same manner as we do Tens from Tens, or Ones from Ones.

EXERCISES FOR THE SLATE AND BOARD.

549	763	452	985	691	826	398	745
321	245	147	756	243	518	289	245

LESSON XLIII.

Carrying every 10 Tens to the Left as 1 Hundred.

EXAMPLE.—Find the Sum of 574 and 253.

EXPLANATION.—Adding the Ones, we write their Sum, 7. Adding the Tens, their Sum is 12 Tens; or 10 Tens and 2 Tens. We write the 2 Tens under the column of Tens. Since the 10 Tens are 1 Hundred, we *carry them to the left* as 1 Hundred, and add this Hundred with the other Hundreds, in the same manner as heretofore, in adding Tens and Ones, we have *carried to the left* every 10 Ones, from the column of Ones, and added them as 1 Ten with the other Tens at the left. Adding 2 Hundreds, 5 Hundreds, and *the 1 Hundred which we carried to the left,* we write 8 under the column of Hundreds.

ADDITION.

$$\left.\begin{array}{r}574\\ \underline{253}\end{array}\right\}\ \textit{Parts.}$$
$$827 \quad \textit{Sum.}$$

Thus we find the Sum of 574 and 253 to be 827.

EXERCISES FOR THE SLATE AND BOARD.

I.

235	162	421	324	142	170	214	362
142	231	235	132	250	218	152	123
231	354	162	261	134	221	451	154

II.

152	215	315	173	219	123	324	218
237	143	267	326	102	257	153	172
123	374	184	184	391	164	247	154
132	125	212	115	173	231	141	431

III.

173	214	365	143	237	378	215	132
254	175	138	375	114	152	171	269
135	143	243	114	345	125	124	171
156	257	156	252	103	261	453	183

LESSON XLIV.

SUBTRACTION.

Carrying 1 Hundred to the Right as 10 Tens.

EXAMPLE. Subtract 379 from 652.

EXPLANATION. *Carrying to the right* 1 of the 5 Tens in the Minuend, and subtracting 9 Ones from 10 Ones and 2 Ones, we have 3 Ones left. Hence we write 3 under the column of Ones.

SOLUTION.

652 *Minuend.*
379 *Subtrahend.*
273 *Difference.*

Having *carried to the right* 1 of the 5 Tens in the Minuend, there are only 4 Tens left. Since we can not subtract 7 Tens from 4 Tens, we *carry to the right* 1 of the 6 Hundreds in the Minuend, and, calling it 10 Tens, subtract the 7 Tens from the 10 Tens and 4 Tens, saying: 7 Tens from 10 Tens leave 3 Tens; 3 Tens and 4 Tens are 7 Tens. We then write 7 Tens.

Having *carried to the right* 1 of the 6 Hundreds in the Minuend, we now subtract the 3 Hundreds in the Subtrahend from the 5 Hundreds left in the Minuend, and write 2 Hundreds in the Difference.

Hence the Difference between 379 and 652 is 273.

EXERCISES FOR THE SLATE AND BOARD.

I.

576	765	648	429	946	328	724	514
234	284	257	156	493	254	251	127

II.

842	578	421	627	724	218	327	876
257	259	156	253	468	104	139	588

III.

711	526	410	674	836	325	939	575
524	389	215	285	647	147	756	197

LESSON XLV.

Exercises for the Slate and Board.

Addition.

I.

158	189	176	184	176	154	187	197
176	147	124	117	145	238	176	137
167	135	189	269	179	197	198	167
134	178	165	175	186	286	184	147
179	186	248	236	179	124	153	157

II.

123	134	125	136	137	148	129	148
123	134	135	116	117	128	119	128
123	134	125	126	147	118	139	118
123	134	115	116	127	138	159	138
123	134	135	136	117	158	119	158
123	134	115	126	137	138	139	118
123	134	125	136	117	128	149	121

Addition at Sight.

9	7	4	8	2	6	9	9	9	9	5	9	9
9	9	9	9	9	9	4	8	2	6	9	7	3

Subtraction.

I.

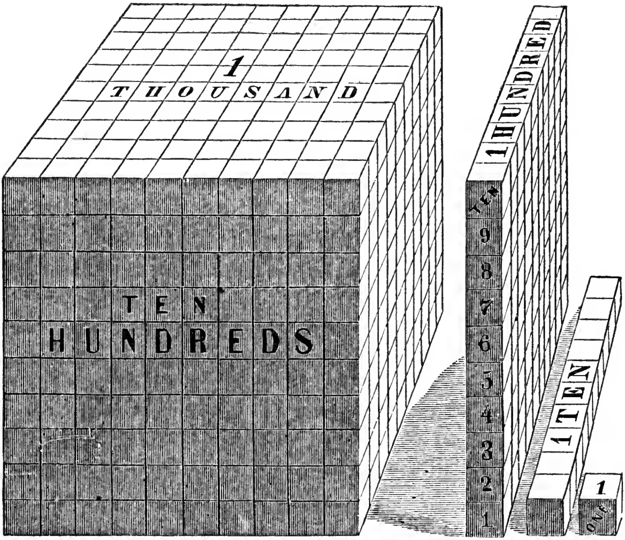
524	743	621	812	546	954	726	314
173	429	238	734	268	267	459	198

II.

957	742	525	326	813	640	435	293
243	529	389	178	508	239	276	108

Subtraction at Sight.

9	9	8	9	8	10	9	10	9	8	10	10
2	4	3	3	5	5	6	8	5	6	6	4



ONE THOUSAND ONE HUNDRED AND ELEVEN;
or, written in figures,

1,111.

LESSON XLVI.

A mechanic sawed out a basketful of Alphabet Blocks and gave them to his little son to count.

The boy first counted them and arranged them as shown in the above cut. Then, turning to his father, he said: "I cannot count beyond Ten. I first counted the blocks in groups having Ten blocks in each. Each of these groups I call a *Ten-group*. After making as many Ten-groups as I could I had 1 single block left, which I have placed by itself, at the right hand.

"I next counted these Ten-groups in the same manner as I counted the single blocks; putting *Ten* Ten-

groups together, and placing them one Ten-group above another; thus making of every *Ten* Ten-groups 1 *larger* group.

"After making as many as I could of these *larger* groups, I had 1 Ten-group remaining, which I have placed *at the left* of the single block."

"Each of these *larger* groups is named a HUNDRED; and the 1 Ten-group and the single block, taken together, are named eleven," remarked the father.

"Then," said the son, "I will call each of these *larger* groups a HUNDRED-GROUP. I then counted my Hundred-groups, placing Ten of them side by side, to make 1 *very large* group. I had 1 Hundred-group remaining, which I placed *at the left* of the 1 Ten-group. *At the left* of this Hundred-group I placed the 1 *very large* group."

Said the father: "This *largest* group is named a ***Thousand.*** Can you tell me how many blocks there are, and then write the number by *figures?*"

The son, after reflecting a moment, promptly replied: "There are ONE THOUSAND ONE HUNDRED AND ELEVEN blocks. I have *one* single block, standing alone, and also *one* group of each kind standing by itself. I will write a figure 1 to stand for the *one* block, and also a figure 1 to stand for each *one* group of the different kinds; writing the figures in the same order as the groups are placed; thus, 1111."

The boy answered correctly. And always in writing a number having Thousands, we write a figure showing the number of Thousands, placing it *at the left* of the figure written for Hundreds.

Read the following numbers:

1111; 1119; 1110; 1211; 1201; 1200; 1000; 5009; 5900; 5990; 7080; 5017; 1001; 9999.

LESSON XLVII.

NOTATION AND NUMERATION.

Writing numbers in figures is named **Notation.**

When numbers are written in *figures, reading them in words* is named **Numeration.**

Read, or Numerate, the following numbers:

1560, 1506, 1056, 156, 1500, 1050, 1005, 7089, 9090, 9017, 1921, 5786, 1870.

Write in figures the following numbers:

1. Three Thousand Five Hundred and Seventy-six;
2. One Thousand Two Hundred and Ten;
3. Two Thousand One Hundred and Three;
4. Four Thousand and Thirty;
5. Five Thousand and Thirteen;
6. Seven Thousand and Three;
7. One Thousand Two Hundred.

Addition.

In adding Hundreds and Thousands, we *carry to the left* every 10 Hundreds, calling them 1 Thousand, and add this Thousand with those in the column of Thousands.

Exercises for the Slate and Board.

1542	1020	2153	1450	1000	2157	1740
2176	1507	1208	1234	2000	1528	2031
1628	2418	1759	2357	2060	1372	1507
3473	1256	1080	1567	1007	2143	1423
1024	2569	3417	1728	3000	1726	2158

Addition at Sight.

8	5	9	6	7	9	7	7	8	8	8	8	9
5	9	6	8	6	7	7	8	6	8	9	7	9

LESSON XLVIII.

SUBTRACTION.

In Subtraction, whenever the Hundreds in the Subtrahend are more than those above them in the Minuend, we *carry to the right* 1 of the Thousands in the Minuend, calling it 10 Hundreds, in the same manner as we have heretofore *carried to the right* 1 Hundred, calling it 10 Tens.

EXERCISES FOR THE SLATE AND BOARD.

I.

5497	7421	9354	3257	8542	6215	9666
2145	5176	5927	1476	4716	4389	5999

II.

4571	5274	7345	9876	6721	2925	8007
2786	1548	5456	4894	3834	1673	5002

Addition.

1439	1247	1020	1300	1172	1526	1000	1210
1256	1072	1513	1040	1094	1017	1200	1030
1073	1364	1327	1000	1207	1432	2030	1140
1142	1289	1156	1708	1165	1157	1005	1327
1381	1070	1208	1159	1082	1215	1234	1456
1574	1528	1097	1023	1521	1000	1357	1579

Addition at Sight.

The *right-hand figure* in the Sum will be *unchanged* so long as the *right-hand figures* in the numbers to be added remain *unchanged,* however we vary the Tens. Thus; 3 and 5 are 8; 13 and 5 are 18; 13 and 15 are 28. *Changing the Tens* in the numbers to be added *changes the Tens* in the Sum, but *not the Ones.*

1	11	11	2	12	12	1	11	11	2	12	12
2	2	12	2	2	12	3	3	13	3	3	13

LESSON XLIX.

RELATION BETWEEN ADDITION AND SUBTRACTION.

EXAMPLE 1. What is the Sum of 347 and 456? Adding as in other cases the Sum is 803.

ADDITION.

347	Two
456	*Numbers.*
803	*Sum.*

EXAMPLE 2. The Sum of two numbers is 803, and one of them is 456; what is the other number?

SUBTRACTION.

Sum.	803	*Minuend.*
One Number.	456	*Subtrahend.*

EXPLANATION. We notice that the Minuend is like the Sum just obtained by Addition; and also that the Subtrahend is one of the two numbers just added. By subtracting we shall obtain, for the Difference, the other of the two numbers added.

Since we cannot take 6 Ones from 3 Ones, we seek in the column of Tens 1 Ten to *carry to the right*, but find none. We will go back to our work in Addition and find the reason for this.

Adding 6 Ones and 7 Ones, we had for the Sum 13 Ones; or 10 Ones and 3 Ones. We wrote the 3 Ones, and *carried* the 10 Ones, as 1 Ten, *to the left*. Adding the 5 Tens, 4 Tens, and the 1 Ten brought from the column of Ones, we had 10 Tens. These 10 Tens we *carried to the left*, as 1 Hundred, and wrote *no Tens*. One of these Tens came from the column of Ones. Thus, in *adding* 456 and 347, we carried 10 Ones not only to the column of Tens, but afterwards, with the 9 Tens, to the column of Hundreds. Now, when we come to *Subtract* 456 from this Sum, 803, to obtain the other number, we cannot *Subtract* until after seeking our 10

Ones, and also our 9 Tens, in the column of Hundreds, *where we left them,* and *carrying them back to the right.*

We take 1 of the 8 Hundreds, and, carrying it to the column of Tens, separate it into 9 Tens and 1 Ten; and leaving the 9 Tens in the column of Tens, *from which we formerly carried them,* we then carry the 1 Ten to the column of Ones, from which it had been carried, and write it as 10 Ones. Subtracting 6 Ones from 10 Ones and 3 Ones, 5 Tens from 9 Tens, and 4 Hundreds from 7 Hundreds, the Difference is 347, the other number.

SOLUTION.

7 9 10 ~~8~~ 0 3	*Minuend re-arranged.*
4 5 6	*Subtrahend.*
3 4 7	*Difference.*

INFERENCES.

I.—If we *unite* two numbers by ADDITION, we may *disunite* them by SUBTRACTION.

II.—If, in *adding* two Numbers, we CARRY TO THE LEFT, in *Subtracting* either Number from the Sum to find the other, we MUST CARRY TO THE RIGHT *the same Numbers* which we *carried to the left in adding.*

EXERCISES FOR THE SLATE AND BOARD.

	1.	2.	3.	4.
Addition:	4526 1375	2746 5273	3254 1647	3127 4375
Subtraction:	5901 1375	8019 5273	4901 1647	7502 4375

	5.	6.	7.	8.
Addition:	1542 3486	6849 2053	1872 4029	3529 4490
Subtraction:	5028 1542	8902 6849	5901 1872	8019 3529

LESSON L.

By turning now to page 68, you will see that the Thousand-group of blocks is in the form of a cube; being 10 blocks in hight, in width, and in length. It contains One Thousand blocks.

This same mechanic made Alphabet Blocks in large quantities, and used to pile them up in cubical Thousand-groups. Then, wrapping a strong paper around each group, he tied up the Thousand blocks firmly in a package.

Having a very large number of such packages on hand one day, he requested his little son and one of the workmen to count them, with the blocks previously counted by the boy.

They counted the packages in Ten-groups; then counted the Ten-groups by Tens, to make Hundred-groups; then counted these Hundred-groups by Tens, to make Thousand-groups.

They found that there were *just as many packages* now as there were *single blocks* formerly counted by the boy; as shown on page 68. They were piled up and arranged in the same manner. At the right was 1 single package; at the left of this 1 Ten-group; next 1 Hundred-group; and last, at the left, 1 Thousand-group.

They now placed their single package at the left of the blocks previously counted by the boy, close by the side of his Thousand-group of blocks, which they tied up as one package. At the left of these they placed their Ten-group of packages; at the left of this their Hundred-group of packages; and last their Thousand-group of packages.

The blocks and groups then stood thus:

CUBE OF PACKAGES.	PACKAGES.			BLOCKS.		
One Thousand-group of Packages.	One Hundred-group.	One Ten-group.	Two Packages.	One Hundred-group.	One Ten-group.	One Block.

Said the boy: "The blocks not in packages stand at the right, and are One Hundred and Eleven. We will write the number of these thus: 111."

The man replied: "Our packages, like our blocks, are Cubes. And since we have *counted* and *arranged* them in the same manner and order as we did the blocks, we will also *write the figures* showing their number in the same manner and order as we did those showing the number of single blocks. We have 2 packages, 1 Ten-group of packages, and 1 Hundred-group of packages; or, One Hundred and Twelve packages. Since these stand at the left of our 111 blocks, we will write the figures 112 at the left of 111; placing a mark like a comma between the two groups of figures; thus, 112,111. Each of these 112 packages contains a Thousand blocks. Hence the figures 112,111 are read: One Hundred and Twelve Thousand One Hundred and Eleven.

"Since we have 1 very large cube, containing One Thousand packages, or 10 Hundred-groups of packages, which stands at the left of our 112 packages, we will write for this a figure 1, placing it at the left of 112, with a point after it; thus, 1,112,111.

"This very large cube of packages is named a ***Million.*** A *Thousand* is a large cube containing a *Thou-*

sand blocks. A *Million* is a very large cube containing a *Thousand Thousands.*"

If, now, we had more Millions, as, for instance, Three Hundred and Sixty-five Millions, instead of One Million, we would write them by placing the figures 365 at the left of 112,111; thus, 365,112,111. We see that, commencing at the right, we have separated the figures of this number into groups each having *three figures.* Each group of figures is named a ***Period.*** The *point,* made like a comma, separating the Periods from each other, is named a ***Period-point.***

The manner of separating a number having nine figures into Periods, and naming the Periods and the Subdivisions in each, is shown in the following

TABLE.

THREE PERIODS.

Numbered,	3d.			2d.			1st.		
Named,	Millions.			Thousands.			Ones.		
Subdivided into and Named,	Hundred-Millions.	Ten-Millions.	Millions.	Hundred-Thousands.	Ten-Thousands.	Thousands.	Hundreds.	Tens.	Ones.
	3	*6*	*5* ,	*1*	*1*	*2* ,	*1*	*1*	*1*

All the Periods are built up in the same manner, each from *its own cube.* Since the cube from which the right-hand Period is built up is *One,* or *One* block, this Period is named the ***Period of Ones.*** The middle Period, having a *Thousand* for its cube, is named the ***Period of Thousands.*** For a like reason the left-hand Period is named the ***Period of Millions.***

LESSON LI.

NOTATION AND NUMERATION.

TABLE.

		1 One	is written	1
10 Ones	are	1 Ten;	written	10
10 Tens	"	1 Hundred;	"	100
10 Hundreds	"	1 Thousand;	"	1,000
10 Thousands	"	1 Ten-thousand;	"	10,000
10 Ten-thousands	"	1 Hundred-thousand;	"	100,000
10 Hundred-thousands	"	1 Million;	"	1,000,000
10 Millions	"	1 Ten-million;	"	10,000,000
10 Ten-millions	"	1 Hundred-million	"	100,000,000

To write numbers in figures: *Commence at the left and write the periods, one after another, in the same order as the words are written.*

Write in Figures the following:

1. Five Thousand; Four Hundred; Seven Thousand and Twenty; Nine Thousand and Seven; Thirteen Thousand, and Eleven.

2. One Hundred and Three Thousand, One Hundred and Seventeen; Three Hundred Thousand, and Twenty-five; Five Millions, One Hundred and Eleven Thousand, and Seventeen.

To read, or numerate, a number written in figures:
1st. Beginning at the right, separate the nvmber into periods;

2d. Beginning at the right, name all the periods;

3d. Beginning at the left, read the periods in order, giving the name of every period except that at the right.

Numerate the following Numbers:

137256984; 40576402; 170510001; 103520407;
19030040; 21005000; 100301200; 31001501.

LESSON LII.

NUMERATION AND ADDITION.

First numerate the numbers in each of the following Exercises, and then find their Sum.

EXERCISES FOR THE SLATE AND BOARD.

I.

35467	127508	1234567	10154125	12536421
12243	105623	2514360	12253326	12037058
10516	100510	1602507	15107908	10250870
27638	209018	1019015	11015017	20900500

II.

32507	234126	3509215	27514310	12141871
10325	576387	5280317	41310080	27423612
47018	154218	4052001	60040085	12751423
53106	627324	6003200	21005002	20205602
61007	542987	1100005	13241576	10030050
27589	173238	7020050	11376529	10400800

Notation, Numeration, and Addition.

In each of the following Examples, first write the numbers, then numerate them, and finally add them.

EXERCISES FOR THE SLATE AND BOARD.

I. Twenty thousand, One Hundred and Five; Thirteen Thousand, and Fifteen; Seventeen Thousand, and Nine.

II. One Hundred and Twenty-five Thousand, Three Hundred and Eleven; Three Hundred and Seven Thousand, Five Hundred and Four; Five Hundred and Eleven Thousand, and Fifteen.

III. Three Million, Five Hundred and Twenty-five Thousand, One Hundred and Twenty-Seven; Five Million, Three Hundred and Seven Thousand, Seven Hundred and Eight; Nine Million, Five Thousand, and Six.

LESSON LIII.

SUBTRACTION.

I.

867,542	5,473,207	63,521,365	750,319,476
573,814	1,632,154	21,507,193	473,207,528

II.

952,541	2,507,318	58,390,504	972,548,765
584,763	1,498,173	28,279,371	481,395,976

Addition and Subtraction.

WRITTEN EXERCISES.

1. Three grain dealers purchased wheat as follows: A purchased 59,487 bushels; B, 47,376; and C, 39,894.

(*a.*) How many bushels did the three men purchase?

(*b.*) How many bushels did A and B together purchase more than C?

2. A father divided his property among his children as follows: He gave to Charles,17,510 dollars; to Henry, 21,437 dollars; to William, 25,196 dollars; to Amelia, 13,087 dollars; to Sarah, 15,193 dollars; and to Susan, 11,981 dollars.

(*a.*) How many dollars did he give to his three sons?

(*b.*) How many dollars to his three daughters?

(*c.*) How many dollars, in all, to his children?

(*d.*) How many more dollars were given to his sons than to his daughters?

3. In the year 1850, the population of the city of New York was 515,547; Boston, 136,881; Philadelphia, 340,045.

(*a.*) What was the total population of the three cities?

(*b.*) How much did the population of New York exceed that of both Philadelphia and Boston?

LESSON LIV.

NUMERATION AND ADDITION.

First read, or numerate, the numbers, and then find their Sum, in each of the following

Exercises for the Slate and Board.

32,507	234,126	3,509,215	27,514,310	12,418,712
10,325	576,387	5,280,317	41,310,080	27,236,129
47,018	154,218	4,052,001	60,040,085	12,514,237
53,106	627,324	6,003,200	21,005,002	20,056,025
61,007	542,987	1,100,005	13,241,576	10,300,500
27,589	173,238	7,020,050	11,376,529	10,008,006

Notation and Addition.

In each of the following Examples, first write the numbers, then numerate them, and finally find the Sum.

Examples for the Slate and Board.

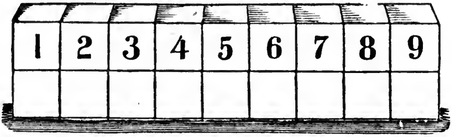
I. Twenty Thousand, One Hundred and five; Thirteen Thousand, and fifteen; Seventeen Thousand, and Nine.

II. One Hundred and Twenty-five Thousand, Three Hundred and Eleven; Three Hundred and Seven Thousand, Five Hundred and Four; Five Hundred and Eleven Thousand, and Fifteen.

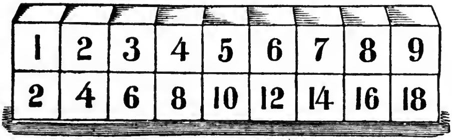
III. Three Million, Five Hundred and Twenty-four Thousand, One Hundred and Twenty-seven; Five Million, Three Hundred and Seven Thousand, Seven Hundred and Eight; Nine Million, Five Thousand, and Six.

IV. Two Hundred and Seventy-six Million, Three Hundred and Seven Thousand, One Hundred and Nineteen; One Hundred and Two Million, Forty-five Thousand, and Twelve; Five Hundred Million, Eight Thousand, and Nine.

LESSON LV.



1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18



These first two horizontal rows of cubes were begun at the left with *one* cube in each, and lengthened by adding another cube to each from time to time, till each row had 9 cubes. The cubes in the upper row are numbered.

Below the figure 1 written on the first cube in the upper row we have written two figure 1's, and have written their Sum below, to show how many cubes there were in *both* rows when there was 1 cube in *each* row.

Under the 2 written in the upper row we have written two figure 2's, and below them their Sum, to show that there were 4 cubes in *both* rows when there were 2 cubes in *each* row.

In the same manner we have found the number of cubes in the 2 rows at each point from their commencement till their completion.

In the second cut we have written these Sums on the cubes in the lower row. We see that when there were 3 cubes in *each* row there were 6 cubes in *both* rows; when there were 4 cubes in *each* row there were 8 in *both* rows.

When the numbers added to form a Sum are *equal*, we name the Addition ***Graded Addition,*** since the Sum is composed of *equal* or *graded* parts.

Since we found these Sums by adding two 1's, two 2's, two 3's, &c., it is plain that the Sums show how many 2 times 1, 2 times 2, 2 times 3, &c., are. Hence, our

GRADED ADDITION TABLE.

2 Times

1 are 2,	4 are 8,	7 are 14,
2 are 4,	5 are 10,	8 are 16,
3 are 6,	6 are 12,	9 are 18.

When we subtract one number from another not *once only*, but *as many times as possible*, we name this Subtraction ***Graded Subtraction.***

We may also consider these cubes as piled up in vertical columns, each having 2 cubes.

From the first column, or 2 cubes at the left, we can *Subtract 2 cubes* once. From the first 2 columns, or 4 cubes at the left, we can *Subtract 2 cubes* 2 times; from 6 cubes 3 times; from 8 cubes 4 times; and so on. Hence we can make the following, or *First Form of*

GRADED SUBTRACTION TABLE.

2 can be Subtracted from

2	1 Time,	8	4 Times,	14	7 Times,
4	2 Times,	10	5 Times,	16	8 Times,
6	3 Times,	12	6 Times,	18	9 Times.

Since 2 cubes are *contained in* any number of cubes as many times as they can be *Subtracted from* that number of cubes, it is evident that we may have the following, more common, or *Second Form of*

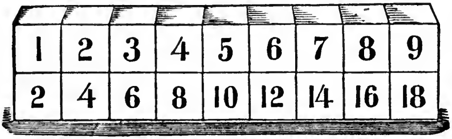
GRADED SUBTRACTION TABLE.

2 are contained in

2	1 Time,	8	4 Times,	14	7 Times,
4	2 Times,	10	5 Times,	16	8 Times,
6	3 Times,	12	6 Times,	18	9 Times.

We can read these Tables from the cubes as a

GRADED ADDITION AND SUBTRACTION TABLE.



To read this as a Graded Addition Table, we commence with the 2 at the bottom, at the left; then take a number in the upper row, as 3; and lastly take, in the lower row, the number below that taken in the upper row, which in this case would be 6; saying: *2 times 3 are 6.* In like manner we proceed, saying: *2 times 4 are 8; 2 times 5 are 10;* and so on.

To read it as a Graded Subtraction Table, we commence with the 2 at the left, as before, but take the two other numbers in a contrary order; reading first the 2 at the left; then a number in the lower row, as 6; and lastly the number 3 above this; saying: *2 are in 6, 3 times.* Thus we go on, saying: *2 are in 8, 4 times; 2 are in 10, 5 times;* and so on.

This form of the Tables, as represented *by the cubes,* is the more convenient.

These Tables should be perfectly learned, and often recited.

LESSON LVI.

Written Mental Exercises.

1. Two horses make 1 span. How many horses are there in 2 spans? 2 times 2 horses are how many?

2. One wagon has 4 wheels. How many wheels have 2 wagons? 2 times 4 wheels are how many wheels?

3. Fanny found 2 birds' nests, each having 6 eggs. How many eggs were there in all? 2 times 6 eggs are how many eggs?

4. In each of 2 windows there are 8 panes of glass. How many panes are there in both windows? 2 times 8 panes are how many?

5. Willie had two rose-bushes with 3 roses on each. How many roses had he on both bushes? 2 times 3 roses are how many roses?

6. Counting my thumbs as fingers, I have 5 fingers on each hand. How many fingers have I on both hands? 2 times 5 fingers are how many?

7. Frank had 7 peaches, and Mary also had 7. How many had both? 2 times 7 peaches are how many?

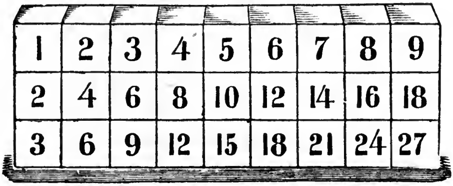
8. Two hens have each 9 chickens. How many chickens have both hens? 2 times 9 are how many?

How many times can we Subtract

2 wagon-wheels from 8 wheels?
2 birds' eggs from 12 eggs?
2 chickens from 18 chickens?
2 peaches from 14 peaches?
2 horses from 4 horses?
2 fingers from 10 fingers?
2 roses from 6 roses?
2 window-panes from 16 panes?

9. How many cents, in all, are 2 times 4 cents and 2 times 3 cents?

LESSON LVII.



In this cut, the 2 horizontal rows of cubes shown in the last cut are seen placed over another row of cubes. Below these cubes numbers are written in vertical columns.

1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
3	6	9	12	15	18	21	24	27

The three 4's, and the 12 written below them, which is their Sum, show that when there were 4 cubes in each of the 3 horizontal rows there were 12 cubes in the 3 rows. So of the other columns of figures and their Sums. These Sums are written on the cubes in the lower row.

The new part added to the Table is read thus as a part of the

GRADED ADDITION TABLE.

3 Times

1 are 3,	4 are 12,	7 are 21,
2 are 6,	5 are 15,	8 are 24,
3 are 9,	6 are 18,	9 are 27.

It is read thus as a part of the

GRADED SUBTRACTION TABLE.

3 can be Subtracted from

3	1 Time,	12	4 Times,	21	7 Times,
6	2 Times,	15	5 Times,	24	8 Times,
9	3 Times,	18	6 Times,	27	9 Times.

Repeat the entire Table; first as a Graded Addition Table, and then as a Graded Subtraction Table.

LESSON LVIII.

Written Mental Exercises.

1. Charles found 3 birds' nests, each having 4 eggs. How many eggs were there in the 3 nests?

2. Emma attended school 5 days each week for 3 weeks. How many days did she attend in 3 weeks?

3. In a school are 3 classes in reading, with 9 pupils in each class. How many pupils in the 3 classes?

4. Each of 3 boys has 8 cents. How many cents have the 3 boys? 3 times 8 are how many?

5. Three boys went fishing, and each caught 7 fish. How many fish did the 3 boys catch?

6. A board fence was 6 boards high. How many boards were there in 3 lengths of the fence?

7. A father had 12 apples, and gave them to his sons, giving each boy 3 apples. How many sons were there? 3 apples are in 12 apples how many times?

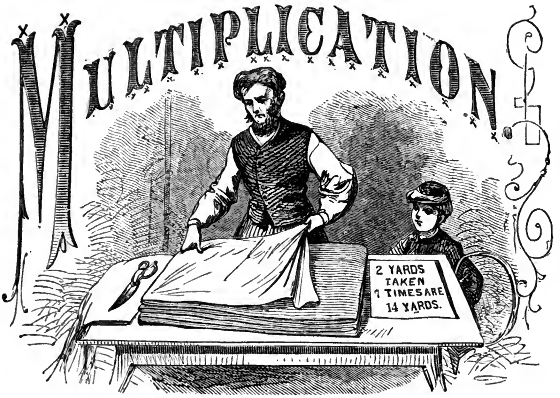
8. A company of girls picked 18 quarts of berries, each girl picking 3 quarts. How many girls were there? 3 are in 18 how many times?

9. Florence had 15 roses in her flower-garden, and there were 3 roses on each bush. How many rose-bushes were there? 3 are in 15 how many times?

10. A beggar received 24 cents from a company of boys, each boy giving him 3 cents. How many boys were there? 3 are in 24 how many times?

11. Willie had 27 cents in 3-cent pieces. How many 3-cent pieces had he? 3 are in 27 how many times?

12. A mother had 21 flowers, and made bouquets of them for her children, putting 3 flowers in each bouquet. How many children had she? 3 are in 21 how many times?



LESSON LIX.

MULTIPLICATION.

Example. A tailor cut from a roll of cloth enough for 7 pairs of boys' pants, using 2 yards for each pair. How many yards did he use?

Explanation. He first measured 2 yards at one end of the roll, and then by *folding* the cloth first one way and then the other, until he had 7 thicknesses, he thus measured 7 times 2 yards.

In thus obtaining 7 times 2 yards, he *folded* the 2 yards *many times.* Taking 2 yards 7 times, and finding how many yards we have, is sometimes named ***Multiplication.*** *Multiplication* means *folding many times,* or *taking many times.* We have taken 2 yards 7 times.

We name the *2 yards* the ***Multiplicand.*** *Multiplicand* means something *to be folded* or *taken many times.*

We name 7 the ***Multiplier.*** The *Multiplier* tells *how many times the Multiplicand is to be taken.*

There are two Solutions for this Example. First: We write a figure 2 *seven times*, and, adding the 2's, write the Sum, 14, below. This method of performing the work we name Graded Addition. Second: We write the 2 *once only*, and then writing a figure 7 under it, to show *how many times the 2 is to be taken*, draw a line under the 7, and saying: "*7 times 2 are 14*," write 14 below the line. This method of performing the work we name *Multiplication*. The result is 14 in both cases; but the 14 obtained by the first Solution is named the *Sum*, and that obtained by the second Solution is named the ***Product***. *Product* means *something produced*. *14 is produced* by multiplying 2 by 7.

First Solution.
By Addition.

$$\left.\begin{array}{r}2\\2\\2\\2\\2\\2\\\underline{2}\end{array}\right\}\begin{array}{c}\textit{Equal}\\\textit{or}\\\textit{Graded}\\\textit{Parts.}\end{array}$$
14 *Sum.*

Second Solution.
By Multiplication.

2 *Multiplicand.*
7 *Multiplier.*
14 *Product.*

Exercises for the Slate and Board.

Graded Addition.

3 Equal	4	20	24	100	124	3,000	3,124
or Graded	4	20	24	100	124	3,000	3,124
Parts.	4	20	24	100	124	3,000	3,124

Multiplication.

I.

Multiplicands:	4	20	24	100	124	3.000	3,124
Multipliers:	3	3	3	3	3	3	3

II.

51,323	93,423	83,213	50,706	90,708
3	2	3	2	3

LESSON LX.

EXAMPLE. How many are 3 times 5?

EXPLANATION. In the first Solution we write 5 *three times*, and, adding, write the Sum, 15; thus, $5 + 5 + 5 = 15$. In the second Solution we write 5 *once only;* and, since it is to be taken *3 times and added*, we write 3 at the right of 5, and, turning the *Sign Plus* thus, $\times$, place it *between* the 5 and 3. The whole stands thus: $5 \times 3 = 15$; and is read: 5 multiplied by 3 equal 15. When the Sign Plus is thus turned and used it does not show that 5 and 3 are to be added together, but shows that 5 are to be taken 3 times and added, to find the Sum; or, which means the same, it shows that 5 are to be multiplied by 3. For this reason, when the Sign Plus is thus turned and used we name it the ***Sign of Multiplication.*** The *Sign of Addition* is a *Vertical Cross.* The *Sign of Multiplication* is an *Inclined Cross.*

FIRST SOLUTION.

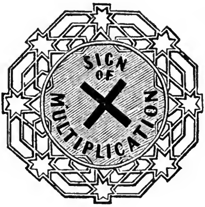
By Addition.

$$5 + 5 + 5 = 15.$$

SECOND SOLUTION.

By Multiplication.

$$5 \times 3 = 15$$



We can write

6 multiplied by 3 equal 18; or $6 \times 3 = 18$;
9 multiplied by 3 equal 27; or $9 \times 3 = 27$.

Write, learn, and recite, the following

TABLE.

$0 \times 2 = 0$	$5 \times 2 = 10$	$0 \times 3 = 0$	$5 \times 3 = 15$
$1 \times 2 = 2$	$6 \times 2 = 12$	$1 \times 3 = 3$	$6 \times 3 = 18$
$2 \times 2 = 4$	$7 \times 2 = 14$	$2 \times 3 = 6$	$7 \times 3 = 21$
$3 \times 2 = 6$	$8 \times 2 = 16$	$3 \times 3 = 9$	$8 \times 3 = 24$
$4 \times 2 = 8$	$9 \times 2 = 18$	$4 \times 3 = 12$	$9 \times 3 = 27$

LESSON LXI.

Example A. Multiply 879 by 3.

Explanation. First; we multiply 9 Ones by 3, and write the Product, 27 Ones. We name this a ***Partial Product,*** because it forms only a *part* of the true Product. Second; we multiply 7 Tens by 3, and, having 21 Tens, or 2 Hundreds and 1 Ten, for our second Partial Product, write 1 Ten in the column of Tens, and 2 Hundreds at the left, in the column of Hundreds. Third; we multiply 8 Hundreds by 3, and, having 24 Hundreds, or 2 Thousands and 4 Hundreds for our third Partial Product, write 4 Hundreds in the column of Hundreds, and 2 Thousands at the left. Now we have taken the 8 Hundreds 3 times, the 7 Tens 3 times, and the 9 Ones 3 times. If we add these three Partial Products we shall have 3 times 879 for the final Product.

SOLUTION.

879	*Multiplicand.*
3	*Multiplier.*
27	*Partial Products.*
21	
24	
2,637	*Product.*

First, we bring down the 7 Ones into the Product. Next, we add 1 Ten and 2 Tens, and write their Sum, 3 Tens, in the Product. Adding 4 Hundreds and 2 Hundreds, we write their Sum, 6 Hundreds, in the Product. Finally, we write 2 Thousands at the left of 6 Hundreds, and have 2,637 for our final Product.

Exercises for the Slate and Board.

I.

57,489	95,847	48,796	85,798	79,846
3	3	3	3	3

II.

64,587	84,758	89,798	94,876	58,764
3	3	3	3	3

LESSON LXII.

EXAMPLE B. Multiply 52,819 by 3.

EXPLANATION. The work in this Solution is placed in nearly the same order as that in the Solution of Example A. The second and third Partial Products are, however, placed in the same horizontal line, since they do not interfere with each other; so also the fourth and fifth.

SOLUTION.

52,819	*Multiplicand.*
3	*Multiplier.*
27	*Partial Products.*
24·3	
15·6	
158,457	*Product.*

EXERCISES FOR THE SLATE AND BOARD.

I.

584,319	769,815	715,394	428,173	936,284
3	3	3	3	3

II.

152,873	231,312	987,546	514,271	859,897
3	3	3	3	3

EXAMPLE C. Multiply 50,608 by 3.

EXPLANATION. Since no *0* in the Multiplicand gives rise to any Partial Product, we write *0* in place of a Partial Product, in such cases.

SOLUTION.

50,608
3
24
18·0
15·0
151,824

EXERCISES FOR THE SLATE AND BOARD.

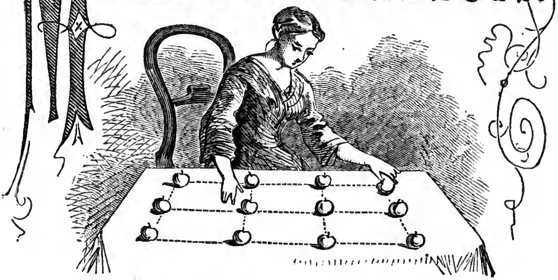
I.

20,708	69,008	304,500	50,980	100,709
2	2	2	2	2

II.

20,030	58,090	600,708	12,500	907,008
3	3	3	3	3

MULTIPLICATION.



LESSON LXIII.

These 12 apples are arranged in two sets of rows; one set running lengthwise of the table, and the other crosswise.

FIRST: If we consider the rows as running *lengthwise*, there are 3 rows. In each row are 4 apples. In 3 rows there are 3 times 4 apples; which are 12 apples. The Multiplicand is 4, the number of apples in each row; the Multiplier is 3, the number of rows; and the Product is 12, the number of apples on the table.

FOUR IN A ROW.

4 *Multiplicand.*
3 *Multiplier.*
12 *Product.*

SECOND: If we consider the rows as running *crosswise*, there are 4 rows. In each row are 3 apples, giving 3 for the Multiplicand. In 4 rows there are 4 times 3 apples; which are 12 apples; giving 4 for a Multiplier, and 12 for the Product.

THREE IN A ROW.

3 *Multiplicand.*
4 *Multiplier.*
12 *Product.*

We notice that the two numbers multiplied together are the same in both cases; and also the Products; but

the Multiplicand and Multiplier of the first change places in the second.

In each case the *Multiplicand shows the number of apples in a row*, the *Multiplier the number of rows*, and the *Product the number of apples arranged.*

INFERENCE. *In all cases where the Product represents* MATERIAL THINGS, *as apples, peaches, oranges, 1st, these things can be so arranged as to stand in two sets of rows; 2d, the number of things in one row of either set may be taken as the Multiplicand, and the number of things in one row of the other set as Multiplier; and, 3d, the number of things arranged will be the Product.*

EXAMPLE. A lady gave 4 apples to each of 3 boys, and 3 apples to each of 4 girls; how many apples did she give to the 3 boys, and how many to the 4 girls?

EXPLANATION. The arrangement of apples in the foregoing cut will give us both answers to this Example.

In the first case there are 4 apples in each row, and 3 rows; or *one row* for each boy. $4 \times 3 = 12$. Hence 12 apples were given to the 3 boys.

In the second case there are 3 apples in each row, and 4 rows; *one row* for each girl. $3 \times 4 = 12$. Hence 12 apples were given to the 4 girls.

3 and 4 multiplied together *make* or *produce* 12. For this reason 3 and 4 are named the ***Factors*** of 12. *Factor* means *Maker* or *Producer*. 12 is the thing *made*, or *produced*, and hence is named the ***Product.*** *Product* means something *made* or *produced*.

From the foregoing Explanations we infer this

PRINCIPLE IN MULTIPLICATION.

If two Numbers are to be multiplied together either may be used as the Multiplicand, and the other as the Multiplier.

LESSON LXIV.

By the use of the Principle stated in the preceding Lesson, we can change the Multiplication Tables already learned into Tables having 1, 2 and 3 as Multiplicands, and 1, 2, 3, 4, 5, 6, 7, 8 and 9, as Multipliers; thus:

FIRST TABLE.	SECOND TABLE.
2 Times 1 are 2,	and 1 Time **2** is 2;
2 Times 2 are 4,	and 2 Times 2 are 4;
2 Times 3 are 6,	and 3 Times 2 are 6;
2 Times 4 are 8,	and 4 Times 2 are 8;
2 Times 5 are 10,	and 5 Times 2 are 10;
2 Times 6 are 12,	and 6 Times 2 are 12;
2 Times 7 are 14,	and 7 Times 2 are 14;
2 Times 8 are 16,	and 8 Times 2 are 16;
2 Times 9 are 18,	and 9 Times 2 are 18.
3 Times 1 are 3,	and 1 Time **3** is 3;
3 Times 2 are 6,	and 2 Times 3 are 6;
3 Times 3 are 9,	and 3 Times 3 are 9;
3 Times 4 are 12,	and 4 Times 3 are 12;
3 Times 5 are 15,	and 5 Times 3 are 15;
3 Times 6 are 18,	and 6 Times 3 are 18;
3 Times 7 are 21,	and 7 Times 3 are 21;
3 Times 8 are 24,	and 8 Times 3 are 24;
3 Times 9 are 27,	and 9 Times 3 are 27.

Learn, and often recite, the above Tables.

Mental Exercises.

$5 \times 2 = ?$	$6 \times 2 = ?$	$7 \times 2 = ?$	$2 \times 6 = ?$
$2 \times 8 = ?$	$8 \times 2 = ?$	$2 \times 8 = ?$	$7 \times 3 = ?$
$3 \times 4 = ?$	$3 \times 6 = ?$	$3 \times 8 = ?$	$8 \times 3 = ?$
$2 \times 5 = ?$	$2 \times 7 = ?$	$2 \times 9 = ?$	$9 \times 2 = ?$
$3 \times 9 = ?$	$3 \times 5 = ?$	$9 \times 3 = ?$	$3 \times 7 = ?$

LESSON LXV.

Exercises for the Slate and Board.

I.

2,132	3,213	1,323	2,131	3,231	2,332
4	5	6	7	8	9

II.

30,201	21,032	23,103	31,232	13,233	33,232
4	5	6	7	8	9

Multiply 31,213,210 by 4; by 5; by 6; by 7; by 8.
Multiply 23,131,032 by 4; by 5; by 6; by 7; by 8.
Multiply 12,302,313 by 4; by 5; by 6; by 7; by 8.
Multiply 30,131,231 by 4; by 5; by 6; by 7; by 9.
Multiply 21,312,103 by 4; by 5; by 6; by 7; by 9.

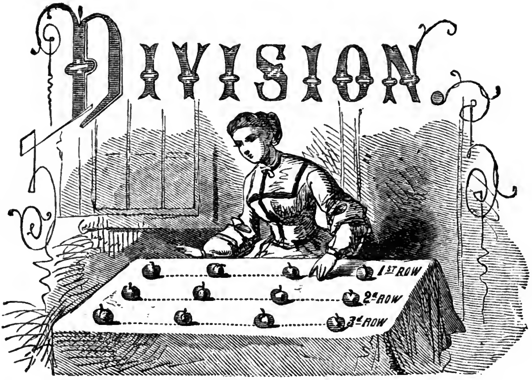
LESSON LXVI.

Exercises for the Slate and Board.

Multiply 31,020,230 by 3; by 4; by 5; by 6; by 7.
Multiply 12,130,302 by 3; by 4; by 5; by 6; by 7.
Multiply 31,213,023 by 3; by 4; by 5; by 6; by 7.
Multiply 21,302,013 by 3; by 4; by 5; by 6; by 8.
Multiply 30,130,231 by 3; by 4; by 5; by 6; by 8.

Multiplication at Sight.

2 × 2	3 × 3	4 × 3	4 × 2	5 × 2	3 × 5
3 × 2	3 × 4	2 × 3	2 × 4	2 × 5	5 × 3
7 × 2	3 × 7	8 × 2	3 × 8	2 × 8	8 × 3
6 × 3	6 × 2	2 × 7	3 × 6	9 × 3	2 × 9
2 × 6	3 × 9	7 × 3	9 × 2	0 × 8	9 × 0



LESSON LXVII.

BOTH NUMBERS STANDING FOR THINGS OF THE SAME KIND.

EXAMPLE.—Harry's mother gave 12 apples to her children, giving 4 apples to each. How many children had she?

EXPLANATION. — 1st: Subtracting 4 apples from 12 apples, we have 8 apples left. These 4 apples are arranged in the *1st row* in the picture. 2d: Subtracting 4 apples from 8 apples, we have 4 apples left. The 4 apples last subtracted are arranged in the *2d row.* 3d: Subtracting 4 apples from 4 apples, we find none remaining. These 4 apples are arranged in the *3d row.*

FIRST SOLUTION.

By Graded Subtraction.

12	*apples.*	*Minuend.*
4	"	*1st Subtrahend.*
8		
4	"	*2d* "
4		
4	"	*3d* "
	No apples left.	

In this manner we divide 12 apples into 3 groups, each having *4 apples.* Since *4 apples* were given to each child, the *number of groups* and the *number of children* must have been the *same.* Hence there were 3 children.

Testing the Result.

If 12 apples can be divided into 3 groups, each having 4 apples, then 3 times 4 apples must equal 12 apples. Therefore, we write *three* 4's and add them. Finding the Sum to be 12, we conclude that there must have been 3 groups, with 4 apples in each. Hence there were 3 children.

TEST.

By Graded Addition.

1st	4	*Apples.*
2d	4	"
3d	4	"
Sum,	12	"

SECOND. — We can perform our work by another method, and find the *same result.* We write 4 apples, and, drawing a line below, write 12 apples below this line. We then ask: *How many times* can 4 apples be subtracted from 12 apples? or, *How many times* are 4 apples contained in 12 apples? Finding that 4 are in 12 *3 times,* we write *3* above the 4 apples, to show *how many groups* there are. Since 12 apples can be divided into 3 groups, each having 4 apples, Harry's mother must have had 3 children.

SECOND SOLUTION.

3	*Groups.*
4	*Apples.*
12	"

Testing the Result.

If there are in 12 apples 3 groups, each having 4 apples, then there are in all 3 times 4 apples; which are 12 apples. Since 3 and 4, in the *Solution,* stand over 12 in the same manner as the Multiplicand and Multiplier stand over their Product, we will multiply them together.

Using 3 as Multiplier, the Product is 12 apples. Hence, in 12 apples there are 3 groups, each having 4 apples. Therefore, 3 children received 12 apples.

The method which we have just used, in the Second Solution, is named ***Division.*** 12 is named the ***Dividend,*** 4 the ***Divisor,*** and 3 the ***Quotient.***

1. DIVISION means *dividing.* We have *divided 12 apples.*

2. DIVIDEND means *something to be divided.* 12 is the number *to be divided.*

3. DIVISOR means *divider.* We have used 4 as a *divider* of 12.

4. QUOTIENT means *how many times.* 3 shows *how many times* 4 are contained in 12.

THIRD. There is still another method of Solution.

THIRD SOLUTION.

$12 \div 4 = 3.$

1st: Since 12 apples are to be divided, we write 12. 2d: Since each child is to receive 4 apples, we write 4 at the right of 12. 3d: Since 4 apples are to be *subtracted* from 12 apples, we write the Sign Minus between 12 and 4. 4th: Since 4 is to be subtracted, *not once only,* but *as many times as possible,* we write a dot above the centre of the Sign Minus, and another below it, to show this. 5th: Since we wish to show what number of times this result *equals,* we write the Sign of Equality after the 4. 6th: Since we find that the number of times 4 can be subtracted from 12 *equals 3 times,* we write 3 at the right of the Sign of Equality. 7th: We read this expression thus: *12 divided by 4 equal 3.*

The *Sign* thus made by changing the Sign Minus is named the ***Sign of Division.***

LESSON LXVIII.

By GRADED SUBTRACTION we find how many times one number can be subtracted from another, or is contained in another.

DIVISION is *a short method of performing Graded Subtraction.*

The Graded Subtraction Tables on pages 83 and 85 can be thus read as a

DIVISION TABLE.

2 are in		**3** are in	
2 Once,		3 Once,	
4 2 Times,	12 6 Times,	6 2 Times,	18 6 Times,
6 3 Times,	14 7 Times,	9 3 Times,	21 7 Times,
8 4 Times,	16 8 Times,	12 4 Times,	24 8 Times,
10 5 Times,	18 9 Times.	15 5 Times,	27 9 Times.

MENTAL EXERCISES.

$14 \div 2 = ?$	$9 \div 3 = ?$	$4 \div 2 = ?$	$6 \div 3 = ?$
$18 \div 3 = ?$	$21 \div 3 = ?$	$12 \div 2 = ?$	$8 \div 2 = ?$
$12 \div 3 = ?$	$10 \div 2 = ?$	$15 \div 3 = ?$	$16 \div 2 = ?$
$6 \div 2 = ?$	$18 \div 2 = ?$	$24 \div 3 = ?$	$27 \div 3 = ?$

1. If 2 boys can sit in one seat at school, how many seats will be required for 12 boys?

EXPLANATION. Since 1 seat is required for 2 boys, the number of seats required for 12 boys is equal to the number of times 2 boys are contained in 12 boys. 2 are in 12 6 times. Therefore 6 seats are required for 12 boys.

2. 18 apples were divided among a company of boys. Each boy had 2 apples. How many boys were there?

3. 2 horses make 1 span. How many spans will 16 horses make?

LESSON LXIX.

THE TWO GIVEN NUMBERS STANDING FOR THINGS UNLIKE IN KIND.

It often happens that the things for which the Dividend and the Divisor stand are *unlike in kind.*

EXAMPLE. Harry's mother gave 12 peaches to her 3 children, Albert, Harry and Walter, giving each the same number. How many peaches did each receive?

EXPLANATION. The *Dividend* is *12 peaches;* and the number *3*, which stands for *boys*, is the *Divisor*. 1st: We take 3 peaches from 12 peaches, and place them on the table, in a row, headed "1st 3." Each boy has *one* of these 3 peaches. 2d: Finding, by Subtraction, that 3 peaches are contained in 12 peaches *4 times*, we place *4 rows* on the table.

SOLUTION.
By Subtraction.

12	*Peaches.*	
3	"	*1st Row.*
9	"	
3	"	*2d* "
6	"	
3	"	*3d* "
3	"	
3	"	*4th* "
No Peaches.		

Each boy has 1 peach in each row, or 4 peaches in all.

But we can consider the rows running crosswise of these. Then each boy's peaches will stand in a row. There will be 3 rows; one row for each boy, with 4 peaches.

In this Solution, 1st: We divide the Dividend, 12 peaches, into parts, each having 3 peaches, and find that there are 4 parts, or rows. 2d: We then consider the rows running crosswise of the first set, and find there are 3 parts, or rows, with 4 peaches in each.

We *divide* by 3 *in the same manner* as we would if it stood for *peaches*. In the end, however, the *Divisor*, 3, is made to stand for *the number of parts*, or rows, into which we divide the Dividend; and the *Quotient*, 4, shows *the size of each part* of the Dividend. Hence,

SOLUTION.
By Division.

4 *Quotient.*
3 *Divisor.*
12 *Dividend.*

Principle 1.

When the DIVISOR *shows the* NUMBER OF PARTS *into which the Dividend is divided, the* QUOTIENT *shows* THE SIZE OF EACH PART *of the Dividend.*

Principle 2.

When the DIVISOR *shows* THE SIZE OF EACH OF THE PARTS *into which the Dividend is divided, the* QUOTIENT *shows* THE NUMBER OF THE PARTS.

Principle 3.

If we regard both the DIVISOR *and* QUOTIENT *in any Example in Division,* ONE OF THEM *will show the* NUMBER OF PARTS *into which the Dividend is divided, and* THE OTHER *the* SIZE OF EACH PART.

LESSON LXX.

MULTIPLICATION.

EXAMPLE 1. Willie's mother gave 5 apples to each of her 3 children. How many apples did she give in all?

EXPLANATION. 1st. Our Product, 15, shows *the whole number* of apples given. 2d. Our Multiplier, 3, shows *the number of parts,* or groups, into which the 15 apples were divided. 3d. Our Multiplicand, 5, shows *the size of each of the parts,* or groups, into which the 15 apples were divided, in giving them to the 3 children.

SOLUTION.

5	*Multiplicand.*
3	*Multiplier.*
15	*Product.*

DIVISION.

EXAMPLE 2. Willie's mother divided 15 apples among her 3 children, giving each the same number. How many apples did she give each child?

EXPLANATION. Writing 15 as the Dividend, and 3 as Divisor, and proceeding as in the Solution on page 101, the Quotient is 5 apples.

SOLUTION.

5	*Quotient.*
3	*Divisor.*
15	*Dividend.*

Examining these two Solutions, we observe:

1st. **15,** *which shows* THE NUMBER OF THINGS DIVIDED, *is the* PRODUCT *in Multiplication and the* DIVIDEND *in Division.*

2d. **3,** *which shows* THE NUMBER OF PARTS *into which 15 things are divided, is the* MULTIPLIER *in Multiplication and the* DIVISOR *in Division.*

3d. **5,** *which shows* THE SIZE OF EACH OF THE PARTS *into which 15 things are divided, is the* MULTIPLICAND *in Multiplication and the* QUOTIENT *in Division.*

4th. The SAME THREE NUMBERS *occur in both Solutions.*

INFERENCES.

1st. The PRODUCT *in Multiplication may be taken as* DIVIDEND.

2d. The MULTIPLIER *in Multiplication may be taken as* DIVISOR.

3d. If the Product be taken as Dividend, and the Multiplier as Divisor, the QUOTIENT *in Division will be the same as the* MULTIPLICAND *in Multiplication.*

4th. If the DIVISOR *and* QUOTIENT BE MULTIPLIED TOGETHER, THE RESULT WILL EQUAL THE DIVIDEND.

Principle 4.

Division consists in finding A NUMBER *which,* WHEN MULTIPLIED BY ONE OF TWO GIVEN NUMBERS, *will give a* PRODUCT EQUAL TO THE OTHER *given number.*

TESTING THE QUOTIENT.

After obtaining our *Quotient,* we can always *test* it by *Inference 4. Multiplying it* by the *Divisor,* if the *result equals the Dividend* we presume that our *Quotient is the true one.*

LESSON LXXI.

Find and test the Quotients in the following

EXERCISES FOR THE SLATE AND BOARD.

Quotients.	6	?	?	?	?	?	?	?	?	?
Divisors.	2	3	2	3	2	3	2	3	2	3
Dividends.	12	18	14	15	18	21	16	27	10	24

Multiplication.

EXAMPLE 1. Find the Product arising from multiplying 34 by 2.

SOLUTION.

34 *Multiplicand.*
2 *Multiplier.*
68 *Product.*

EXPLANATION. We multiply first the 4 Ones, and then the 3 Tens, by 2, as in former Examples, and, writing the results in the Product, have 68.

Division.

EXAMPLE 2. Find the Quotient arising from dividing 68 by 2.

EXPLANATION. From "Principle 4," we find that Division consists, in this Example, in finding a number which, when multiplied by 2, will give a result equal to 68.

SOLUTION.

34 *Quotient.*
2 *Divisor.*
68 *Dividend.*
68 *Test Product.*

1st. We find how many Tens multiplied by 2 will give the 6 Tens of the Dividend. Since 3 Tens multiplied by 2 give 6 Tens, we write 3 Tens in the Quotient.

2d. Finding that 4 Ones multiplied by 2 give 8 Ones, we write 4 Ones in the Quotient.

3d. *Testing our Quotient,* we multiply 34 by 2, and write the result 68, as a *Test Product,* below the Dividend.

Find and Test the Quotients in the following

EXERCISES FOR THE SLATE AND BOARD.

I.

Quotients.	?	?	?	?	?	?	?
Divisors.	2	2	2	2	2	2	2
Dividends.	42	64	46	68	48	88	68
Test Products.	42	....	....	....	....	....	....

II.

	?	?	?	?	?	?	?
Divisors.	3	3	3	3	3	3	3
Dividends.	63	66	39	69	93	99	369

III.

468 ÷ 2	684 ÷ 2	2,468 ÷ 2	68,468 ÷ 2
639 ÷ 3	306 ÷ 3	9,630 ÷ 3	60,936 ÷ 3

LESSON LXXII.

MULTIPLICATION.

EXAMPLE A. Multiply 376 by 2.

EXPLANATION. Multiplying 6 Ones, then 7 Tens, and lastly 3 Hundreds, separately by 2, and writing the Partial Products and adding them, we have 752 for our final Product.

SOLUTION.

376	*Multiplicand.*
2	*Multiplier.*
12	*Partial*
14	*Products.*
6	
752	*Product.*

DIVISION.

EXAMPLE B. Divide 752 by 2.

EXPLANATION. 1st. Our Hundreds in the Quotient cannot be more than 3, since 4 Hundreds multiplied by 2 would give 8 Hundreds. Hence we write 3 Hundreds in the Quotient. Multiplying 3 Hundreds by 2, we write the Product, 6 Hundreds, under the 7 Hundreds. Drawing a line below the 6, we subtract 6 from 7, and have 1 Hundred remaining.

SOLUTION.

376	*Quotient.*
2	*Divisor.*
752	*Dividend.*
6	
15	
14	
12	
12	

2d. We now bring down the 5 Tens of the Dividend; and, writing 5 at the right of the 1 Hundred, have 1 Hundred and 5 Tens, or 15 Tens, for a new Partial Dividend. Proceeding to divide this by 2, we see that we cannot have more than 7 Tens in the Quotient. Writing 7 Tens in the Quotient, and multiplying them by 2, we write the Product, 14 Tens, under the 15 Tens. Subtracting, we have 1 Ten remaining.

3d. Lastly, we bring down the 2 Ones of the Dividend; and, writing 2 at the right of the 1 Ten, have 1 Ten and 2 Ones, or 12 Ones. Dividing 12 Ones by 2,

we write 6 Ones in the Quotient. Multiplying 6 Ones by 2, and subtracting, there is *no Remainder.* Hence our entire Quotient is 376.

TESTING THE QUOTIENT.

To Test our Quotient we would multiply it by the Divisor. The work would be the same as in Example A.

From our work in both Examples, we observe, 1st: The Partial Products *in both* are 6 Hundreds, 14 Tens, and 12 Ones; but they stand in *contrary order.* In Example A, we *added* these, and obtained 752. In Example B, we *subtracted* them from 752, in a *reverse order,* and had *no Remainder.* 2d: In Example A, we *multiplied together two numbers,* 376 and 2, and obtained 752 for a *Product;* and in Example B we *divided this Product by one of the numbers,* 2, and obtained, for a *Quotient, the other number,* 376. Hence, we draw the following

INFERENCE.

DIVISION *is precisely the* REVERSE OF MULTIPLICATION.

Principle 5.

If the PRODUCT *of two numbers be* DIVIDED BY ONE *of them, the* QUOTIENT *will be* THE OTHER.

Find and Test the Quotients in the following

EXERCISES FOR THE SLATE AND BOARD.

I.

Quotients.	?	?	?	?	?	?
Divisors.	2	3	2	3	2	3
Dividends.	312	435	538	768	756	867

II.

5,871 ÷ 3	9,786 ÷ 2	7,467 ÷ 3	9,536 ÷ 2
3,976 ÷ 2	8,568 ÷ 3	5,796 ÷ 2	4,125 ÷ 3
7,641 ÷ 3	5,876 ÷ 2	8,085 ÷ 3	1,974 ÷ 2

LESSON LXXIII.

The three numbers will be precisely alike, when obtained, in both Examples of each Pair of the following

Exercises for the Slate and Board.

		1.	2.	3.
Multiplication:		59,758	789,675	376,586
		2	3	2
Division:		2	3	2
		119,516	2,369,025	753,172

By Principle 5 we change our Multiplication Tables into two sets of Division Tables; thus

2		1 are 2,	**2**		2	1 Time,	1	*is in*	2	**2** Times,
2		2 are 4,	2		4	2 Times,	2		4	2 Times,
2		3 are 6,	2		6	3 Times,	3		6	2 Times,
2		4 are 8,	2		8	4 Times,	4		8	2 Times,
2	*Times*	5 are 10,	2	*are in*	10	5 Times,	5	*are in*	10	2 Times,
2		6 are 12,	2		12	6 Times,	6		12	2 Times,
2		7 are 14,	2		14	7 Times,	7		14	2 Times,
2		8 are 16,	2		16	8 Times,	8		16	2 Times,
2		9 are 18,	2		18	9 Times,	9		18	2 Times.

3		1 are 3,	**3**		3	1 Time,	1	*is in*	3	**3** Times,
3		2 are 6,	3		6	2 Times,	2		6	3 Times,
3		3 are 9,	3		9	3 Times,	3		9	3 Times,
3		4 are 12,	3		12	4 Times,	4		12	3 Times,
3	*Times*	5 are 15,	3	*are in*	15	5 Times,	5	*are in*	15	3 Times,
3		6 are 18,	3		18	6 Times,	6		18	3 Times,
3		7 are 21,	3		21	7 Times,	7		21	3 Times,
3		8 are 24,	3		24	8 Times,	8		24	3 Times,
3		9 are 27,	3		27	9 Times,	9		27	3 Times.

LESSON LXXIV.

The equation $5 \times 3 = 15$ formed by *Multiplication,* we name an ***Equation by Multiplication.***

Since the equation $15 \div 3 = 5$ is formed by *Division,* we will name it an ***Equation by Division.***

From *three numbers* such that no two of them are equal, and the greatest equals the Product of the two others, we can form *two Equations by Multiplication,* by the Principle given on page 93, and also *two Equations by Division,* by Principle 5 in Division.

TO FORM TWO EQUATIONS BY MULTIPLICATION:

RULE.

I. *Write the two smaller numbers, with the Sign* $\times$ *between them, for the First Member of an Equation.*

II. *Write the greatest number for the Second Member, placing the Sign* $=$ *between the Members.*

III. *Form the second Equation from the first, by changing the places of the two smaller numbers.*

TO FORM TWO EQUATIONS BY DIVISION:

RULE.

I. *Write the greatest number, and after it one of the other numbers, placing the Sign* $\div$ *between them, for the First Member of an Equation.*

II. *Write the remaining number for the Second Member, placing the Sign* $=$ *between the Members.*

III. *Form the second Equation by Division from the first, by changing the places of the two smaller numbers.*

Form 4 Equations from each of these sets of numbers:

2, 7 and 14; 6, 3 and 18; 2, 8 and 16; 3, 7 and 21;
9, 2 and 18; 3, 8 and 24; 6, 2 and 12; 9, 3 and 27;
5, 3 and 15; 4, 3 and 12; 2, 5 and 10; 4, 2 and 8.

LESSON LXXV.

In the Equation $2 \times 3 = 6$, if 6 were erased, it could be found by *multiplying* together 2 and 3; if 2 were erased, we could find it by *dividing* 6 by 3; and if 3 were erased, it could be found by *dividing* 6 by 2. Hence,

TO REPLACE A NUMBER IN AN EQUATION BY MULTIPLICATION:

RULE.

I. *If the Second Member be missing, find it by* MULTIPLYING *together the two numbers in the First Member.*

II. *If either number in the First Member be missing, find it by* DIVIDING *the* SECOND MEMBER *by the* OTHER NUMBER *in the First Member.*

EXERCISES FOR THE SLATE AND BOARD.

$3 \times ? = 18$	$? \times 5 = 15$	$8 \times 2 = ?$	$6 \times 3 = ?$
$? \times 3 = 12$	$7 \times ? = 14$	$7 \times 3 = ?$	$7 \times ? = 21$
$3 \times 8 = ?$	$3 \times ? = 27$	$? \times 6 = 18$	$2 \times ? = 16$

As we see at the right, the *three numbers* in the Solution of an Example in *Multiplication* and one in *Division* are the *same*, and *stand in the same order*. Hence,

Quotient.	7	*Multiplicand.*
Divisor.	3	*Multiplier.*
Dividend.	21	*Product.*

TO FIND ANY NUMBER IN AN EXAMPLE IN MULTIPLICATION OR DIVISION, WHEN MISSING:

RULE.

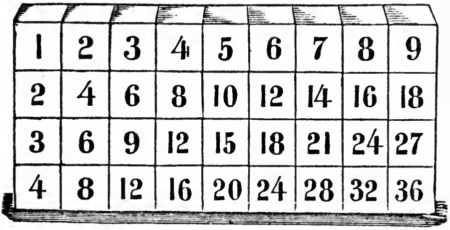
I. *If* EITHER OF THE FIRST TWO, *or smaller numbers, be missing, find it by* DIVIDING *the third, or* GREATEST, *by the* SMALLER NUMBER *which is given.*

II. *If the* THIRD, *or greatest number, be missing, find it by* MULTIPLYING TOGETHER THE TWO OTHERS.

EXERCISES FOR THE SLATE AND BOARD.

5	?	8	?	?	3	3	?	?	9
?	7	?	6	4	?	?	3	8	?
10	14	16	18	12	15	18	21	24	27

LESSON LXXVI.



We now place the 3 horizontal rows of cubes, on page 85, over another row. The portion now added to the Table is formed in the same manner as the portion then made.

From the Multiplication Table given below, and also from each one hereafter given, make a second Table by changing the places of the figures in the first and second columns; thus: 1 time 4 is 4; 2 times 4 are 8; 3 times 4 are 12; &c., &c.

MULTIPLICATION TABLE.

4		1 are	4,
4		2 are	8,
4		3 are	12,
4	*Times*	4 are	16,
4		5 are	20,
4		6 are	24,
4		7 are	28,
4		8 are	32,
4		9 are	36.

DIVISION TABLES.

4		4	1 Time,
4		8	2 Times,
4		12	3 Times,
4	*are in*	16	4 Times,
4		20	5 Times,
4		24	6 Times,
4		28	7 Times,
4		32	8 Times,
4		36	9 Times,

1	*is in*	4	**4** Times,
2		8	4 Times,
3		12	4 Times,
4		16	4 Times,
5	*are in*	20	4 Times,
6		24	4 Times,
7		28	4 Times,
8		32	4 Times,
9		36	4 Times.

The same 3 columns of figures occur in all the 3 Tables on the preceding page.

Mental Exercises.

I.

$9 \times 4 = ?$	$5 \times ? = 20$	$7 \times ? = 28$	$8 \times ? = 32$
$6 \times ? = 18$	$? \times 4 = 24$	$? \times 4 = 28$	$9 \times ? = 36$
$? \times 8 = 32$	$4 \times ? = 28$	$6 \times ? = 24$	$? \times 4 = 36$

II.

$16 \div 4 = ?$	$21 \div 7 = ?$	$28 \div ? = 7$	$? \div 6 = 4$
$21 \div ? = 7$	$20 \div 4 = ?$	$16 \div 4 = ?$	$? \div 4 = 4$
$24 \div 4 = ?$	$? \div 4 = 9$	$? \div 8 = 4$	$36 \div ? = 4$

LESSON LXXVII.

MULTIPLICATION.

I.

587,936	759,864	434,234	342,344
4	4	9	8

II.

241,243	423,132	214,324	879,567
7	6	5	4

III.

Multiply 414,342 by 4; by 5; by 6; by 7; by 8;
Multiply 340,424 by 4; by 5; by 6; by 7; by 8.

DIVISION.

I.

4	3	4	8
7,849,896	8,626,587	4,787,104	2,738,752

II.

7	6	5	4
1,688,701	2,538,726	7,171,060	9,276,872

LESSON LXXVIII.

It should be noticed that the first part of the Tables given below, extending to the line commencing "5 times 4 are 20," has been given in the preceding Tables. The new part is printed in the Italic type.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45

MULTIPLICATION TABLE.		DIVISION TABLES.				
5 Times 1 are 5,	**5** are in 5	1 Time,	1 is in 5	**5** Times,
5 Times 2 are 10,	5 are in 10	2 Times,	2 is in 10	5 Times,
5 Times 3 are 15,	5 are in 15	3 Times,	3 is in 15	5 Times,
5 Times 4 are 20,	5 are in 20	4 Times,	4 is in 20	5 Times,
5 Times *5* are *25*,	*5* are in *25*	*5* Times,	*5* are in *25*	*5* Times,
5 Times *6* are *30*,	*5* are in *30*	*6* Times,	*6* are in *30*	*5* Times,
5 Times *7* are *35*,	*5* are in *35*	*7* Times,	*7* are in *35*	*5* Times,
5 Times *8* are *40*,	*5* are in *40*	*8* Times,	*8* are in *40*	*5* Times,
5 Times *9* are *45*.	*5* are in *45*	*9* Times,	*9* are in *45*	*5* Times.

Write one more Equation by Multiplication and two by Division from each of the following

Equations.

$4 \times 6 = 24$ $5 \times 7 = 35$ $6 \times 5 = 30$ $4 \times 7 = 28$
$8 \times 5 = 40$ $4 \times 8 = 32$ $9 \times 4 = 36$ $9 \times 5 = 45$

MENTAL EXERCISES.

I.

$5 \times 7 = ?$ $4 \times ? = 36$ $5 \times ? = 30$ $5 \times 9 = ?$
$? \times 9 = 36$ $8 \times ? = 40$ $9 \times ? = 27$ $5 \times ? = 45$
$5 \times ? = 40$ $? \times 5 = 25$ $? \times 9 = 45$ $7 \times ? = 35$

II.

$35 \div 5 = ?$	$? \div 7 = 3$	$36 \div 4 = ?$	$36 \div 9 = ?$
$30 \div 5 = ?$	$40 \div 8 = ?$	$28 \div 7 = ?$	$? \div 9 = 5$

III.

5	?	?	9	?	7	5	5	?	9
7	7	4	5	9	?	8	?	5	?
?	35	28	?	45	35	?	30	40	45

LESSON LXXIX.

Exercises for the Slate and Board.

Multiplication.

1. Multiply 874,596 by 2; by 3; by 4; by 5.
2. Multiply 746,789 by 2; by 3; by 4; by 5.
3. Multiply 876,598 by 2; by 3; by 4; by 5.
4. Multiply 715,829 by 2; by 3; by 4; by 5.
5. Multiply 769,854 by 2; by 3; by 4; by 5.

Division.

1. Divide 114,840 by 2; by 3; by 4; by 5.
2. Divide 689,040 by 2; by 3; by 4; by 5.
3. Divide 803,880 by 2; by 3; by 4; by 5.
4. Divide 344,520 by 2; by 3; by 4; by 5.
5. Divide 107,640 by 2; by 3; by 4; by 5.

Multiplication at Sight.

4×4	4×5	6×5	4×7	6×4	7×4
5×5	4×6	5×4	7×5	5×6	3×9
3×8	5×8	8×4	9×3	8×3	4×9

Division at Sight.

$6 \div 2$	$5 \div 5$	$8 \div 4$	$10 \div 2$	$10 \div 5$	$14 \div 2$
$8 \div 2$	$20 \div 5$	$9 \div 3$	$25 \div 5$	$12 \div 2$	$12 \div 6$
$15 \div 5$	$16 \div 4$	$16 \div 8$	$12 \div 3$	$12 \div 4$	$18 \div 2$
$20 \div 4$	$15 \div 3$	$14 \div 7$	$16 \div 2$	$18 \div 3$	$21 \div 3$

LESSON LXXX.

The four Tables given below should be thoroughly learned. They can all be read from the Table of cubes shown at the right. The new part now added is printed in Italic type.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54

MULTIPLICATION TABLE.

6		1 are	6,
6		2 are	12,
6		3 are	18,
6		4 are	24,
6	Times	5 are	30,
6		*6* are	*36*,
6		*7* are	*42*,
6		*8* are	*48*,
6		*9* are	*54*.

DIVISION TABLES.

6		6	1 Time,
6		12	2 Times,
6		18	3 Times,
6		24	4 Times,
6	are in	30	5 Times,
6		*36*	*6* Times,
6		*42*	*7* Times,
6		*48*	*8* Times,
6		*54*	*9* Times,

1	is in	6	**6** Times,
2		12	6 Times,
3		18	6 Times,
4		24	6 Times,
5		30	6 Times,
6	are in	*36*	*6* Times,
7		*42*	*6* Times,
8		*48*	*6* Times,
9		*54*	*6* Times.

Write three other Equations from each of the following

Equations.

$6 \times 7 = 42$ $6 \times 8 = 48$ $6 \times 9 = 54$ $5 \times 6 = 30$
$5 \times 8 = 40$ $7 \times 5 = 35$ $9 \times 4 = 36$ $9 \times 5 = 45$

MENTAL EXERCISES.

I.

$? \times 7 = 42$ $6 \times 8 = ?$ $7 \times ? = 42$ $6 \times ? = 36$
$9 \times ? = 54$ $? \times 8 = 48$ $? \times 6 = 48$ $? \times 6 = 54$

II.

$36 \div 6 = ?$	$54 \div ? = 6$	$25 \div 5 = ?$	$30 \div ? = 5$
$42 \div ? = 6$	$? \div 7 = 6$	$48 \div 6 = ?$	$? \div 8 = 6$
$? \div 9 = 6$	$48 \div 8 = ?$	$? \div 6 = 7$	$54 \div 9 = ?$

III.

?	?	7	8	6	?	8	?	5	9
6	6	?	?	?	6	6	9	?	6
36	42	42	48	48	54	?	54	35	?

LESSON LXXXI.

MULTIPLICATION.

1. Multiply 875,964 by 2; by 3; by 4; by 5; by 6.
2. Multiply 587,958 by 2; by 3; by 4; by 5; by 6.
3. Multiply 978,547 by 2; by 3; by 4; by 5; by 6.
4. Multiply 849,756 by 2; by 3; by 4; by 5; by 6.
5. Multiply 354,623 by 5; by 6; by 7; by 8; by 9.
6. Multiply 560,365 by 5; by 6; by 7; by 8; by 9.
7. Multiply 465,324 by 5; by 6; by 7; by 8; by 9.

Division.

1. Divide 3,232,740 by 2; by 3; by 4; by 5; by 6.
2. Divide 4,765,140 by 2; by 3; by 4; by 5; by 6.
3. Divide 1,854,240 by 2; by 3; by 4; by 5; by 6.
4. Divide 1,683,840 by 2; by 3; by 4; by 5; by 6.
5. Divide 2,918,520 by 2; by 3; by 4; by 5; by 6.

Multiplication at Sight.

4×5	6×4	8×5	3×9	5×8	7×5
6×6	7×6	6×9	5×6	9×5	6×8
6×7	8×6	4×6	5×9	9×6	6×5

Division at Sight.

$42 \div 6$	$36 \div 6$	$45 \div 9$	$54 \div 6$	$48 \div 8$	$40 \div 8$
$35 \div 7$	$54 \div 9$	$48 \div 6$	$42 \div 7$	$30 \div 6$	$45 \div 5$
$36 \div 9$	$35 \div 5$	$30 \div 5$	$32 \div 4$	$40 \div 5$	$25 \div 5$

LESSON LXXXII.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63

MULTIPLICATION TABLE.

DIVISION TABLES.

Multiplication Table			Division Tables						
7	1 are 7,		**7**	7	1 Time,	1 *is in*	7	**7** Times,
7	2 are 14,		7	14	2 Times,	2	14	7 Times,
7	3 are 21,		7	21	3 Times,	3	21	7 Times,
7 *Times*	4 are 28,		7 *are in*	28	4 Times,	4	28	7 Times,
7	5 are 35,		7	35	5 Times,	5 *are in*	35	7 Times,
7	6 are 42,		7	42	6 Times,	6	42	7 Times,
7	*7* are *49*,		7	*49*	*7* Times,	*7*	*49*	7 Times,
7	*8* are *56*,		7	*56*	*8* Times,	*8*	*56*	7 Times,
7	*9* are *63*.		7	*63*	*9* Times,	*9*	*63*	7 Times.

Write three others from each of the following

Equations.

$7 \times 8 = 56$ $\quad 8 \times 6 = 48$ $\quad 6 \times 9 = 54$ $\quad 7 \times 9 = 63$

$35 \div 7 = 5$ $\quad 42 \div 7 = 6$ $\quad 30 \div 6 = 5$ $\quad 36 \div 9 = 4$

Mental Exercises.

I.

$7 \times ? = 42$ $\quad 7 \times ? = 56$ $\quad 7 \times ? = 63$ $\quad 7 \times ? = 49$

$? \times 7 = 63$ $\quad ? \times 7 = 56$ $\quad ? \times 9 = 63$ $\quad 9 \times 7 = ?$

II.

$36 \div 6 = ?$	$56 \div ? = 8$	$42 \div ? = 6$	$63 \div ? = 9$
$49 \div 7 = ?$	$63 \div 7 = ?$	$? \div 7 = 7$	$? \div 7 = 6$
$49 \div ? = 7$	$? \div 8 = 7$	$56 \div ? = 7$	$? \div 9 = 7$

III.

?	9	7	9	?	?	7	?	6
8	?	9	?	7	7	?	8	?
56	54	?	63	56	49	63	48	42

LESSON LXXXIII.

Exercises for the Slate and Board.

Multiplication.

795,869 × 7	978,549 × 7	895,987 × 7
536,754 × 8	765,467 × 8	657,327 × 8
534,276 × 9	374,657 × 9	534,756 × 9
978,679 × 7	576,754 × 8	321,567 × 9

Division.

5,984,678 ÷ 7	9,854,789 ÷ 7	6,897,583 ÷ 7
5,416,040 ÷ 8	5,174,016 ÷ 8	4,996,504 ÷ 8
6,395,148 ÷ 9	3,784,878 ÷ 9	1,918,575 ÷ 9
9,546,327 ÷ 7	3,715,616 ÷ 8	5,982,039 ÷ 9

Divide 544,320 by 4; by 5; by 6; by 7.
Divide 5,987,520 by 4; by 5; by 6; by 7.

Multiplication at Sight.

5 × 6	6 × 6	5 × 3	8 × 4	7 × 4	4 × 7
7 × 7	7 × 8	7 × 9	8 × 7	9 × 7	6 × 7
5 × 7	7 × 6	6 × 5	5 × 9	7 × 5	5 × 5

Division at Sight.

35 ÷ 7	42 ÷ 7	63 ÷ 9	56 ÷ 7	42 ÷ 6	49 ÷ 7
54 ÷ 6	30 ÷ 5	48 ÷ 6	36 ÷ 6	30 ÷ 6	48 ÷ 8
56 ÷ 8	63 ÷ 7	25 ÷ 5	54 ÷ 9	35 ÷ 5	28 ÷ 7

LESSON LXXXIV.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72

MULTIPLICATION TABLE.				*DIVISION TABLES.*						
8		1 are 8,	**8**		8	1 Time,	1	*is in*	8	**8** Times,
8		2 are 16,	8		16	2 Times,	2		16	8 Times,
8		3 are 24,	8		24	3 Times,	3		24	8 Times,
8		4 are 32,	8		32	4 Times,	4		32	8 Times,
8	*Times*	5 are 40,	8	*are in*	40	5 Times,	5	*are in*	40	8 Times,
8		6 are 48,	8		48	6 Times,	6		48	8 Times,
8		7 are 56,	8		56	7 Times,	7		56	8 Times,
8		*8* are *64*,	*8*		*64*	*8* Times,	*8*		*64*	*8* Times,
8		*9* are *72*.	*8*		*72*	*9* Times,	*9*		*72*	*8* Times.

Write three others from each of the following

Equations.

$6 \times 8 = 48 \quad 7 \times 9 = 63 \quad 9 \times 8 = 72 \quad 7 \times 8 = 56$

MENTAL EXERCISES.

I.

$7 \times 8 = ? \quad 8 \times ? = 56 \quad ? \times 6 = 48 \quad 8 \times ? = 64$

$9 \times 8 = ? \quad 9 \times ? = 63 \quad 8 \times ? = 72 \quad ? \times 9 = 72$

II.

$64 \div 8 = ?$ $56 \div ? = 7$ $72 \div 9 = ?$ $? \div 8 = 8$

$? \div 7 = 8$ $72 \div 8 = ?$ $? \div 9 = 8$ $56 \div ? = 8$

III.

7	?	8	9	?	8	7	?	?	8
7	8	?	?	9	9	?	8	8	8
?	56	64	72	63	?	56	72	64	?

LESSON LXXXV.

Exercises for the Slate and Board.

Multiplication.

785,968 × 8	978,786 × 8	879,587 × 8
874,569 × 8	678,587 × 9	768,576 × 9
854,867 × 9	758,675 × 9	795,879 × 8

1. Multiply 859,756 by 5; by 6; by 7; by 8.
2. Multiply 596,873 by 5; by 6; by 7; by 8.
3. Multiply 584,762 by 4; by 7; by 8; by 9.
4. Multiply 378,578 by 4; by 7; by 8; by 9.

Division.

687,952 ÷ 8	598,648 ÷ 8	987,688 ÷ 8
789,872 ÷ 8	759,132 ÷ 9	653,445 ÷ 9
245,439 ÷ 9	753,588 ÷ 9	579,568 ÷ 8

1. Divide 164,304 by 4; by 6; by 7; by 8.
2. Divide 332,472 by 4; by 6; by 7; by 8.
3. Divide 349,440 by 4; by 6; by 7; by 8.
4. Divide 903,504 by 2; by 3; by 4; by 8.
5. Divide 910,680 by 3; by 4; by 5; by 8.

Multiplication at Sight.

8 × 9 7 × 8 8 × 8 7 × 9 9 × 8 6 × 8

Division at Sight.

63 ÷ 9 64 ÷ 8 72 ÷ 8 56 ÷ 8 72 ÷ 9 48 ÷ 8

49 ÷ 7 56 ÷ 7 48 ÷ 6 63 ÷ 7 40 ÷ 5 45 ÷ 9

LESSON LXXXVI.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

MULTIPLICATION TABLE.

9	Times	1 are	9,
9		2 are	18,
9		3 are	27,
9		4 are	36,
9		5 are	45,
9		6 are	54,
9		7 are	63,
9		8 are	72,
9		*9* are	*81*.

DIVISION TABLES.

9	are in	9	1 Time,	1	is in	9	**9** Times,
9		18	2 Times,	2	are in	18	9 Times,
9		27	3 Times,	3		27	9 Times,
9		36	4 Times,	4		36	9 Times,
9		45	5 Times,	5		45	9 Times,
9		54	6 Times,	6		54	9 Times,
9		63	7 Times,	7		63	9 Times,
9		72	8 Times,	8		72	9 Times,

9 are in *81* *9* Times.

Mental Exercises.

I.

$8 \times ? = 56$	$? \times 9 = 63$	$8 \times ? = 72$	$9 \times 9 = ?$
$? \times 9 = 72$	$7 \times ? = 63$	$9 \times 7 = ?$	$9 \times 8 = ?$
$8 \times 8 = ?$	$9 \times ? = 81$	$? \times 8 = 56$	$? \times 9 = 81$

II.

$63 \div ? = 7$	$? \div 9 = 7$	$? \div 9 = 8$	$81 \div ? = 9$
$72 \div 8 = ?$	$81 \div 9 = ?$	$? \div 8 = 8$	$? \div 9 = 9$

III.

?	?	9	9	9	?	9	8	9	?
9	8	?	?	9	9	8	?	?	8
63	72	63	72	?	81	?	72	81	64

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3		9	12	15	18	21	24	27
4			16	20	24	28	32	36
5				25	30	35	40	45
6					36	42	48	54
7						49	56	63
8							64	72
9								81

By comparing the above Table with that given on the preceding page, it will be seen that the numbers omitted in forming this Table from that are the same as those retained. Hence they were unnecessary.

In reading this Table we find our Multiplier, or Divisor, in the upper horizontal row whenever it is greater than the Multiplicand, or Quotient.

LESSON LXXXVII.

EXAMPLE A. Multiply 964 by 3.

EXPLANATION. 1st. The first Solution is in the form heretofore used, the Partial Products being written.

2d. In the second Solution the Partial Products are not written out in full. We first multiply 4 Ones by 3, and, obtaining 12 Ones, or 1 Ten and 2 Ones, for a Partial Product, *write* the *2 Ones*, and *retain* the *1 Ten* in the mind. Next, we multiply the 6 Tens by 3, and, obtaining 18 Tens for a Partial Product, we add to these the 1 Ten retained in the mind, and have 19 Tens, or 1 Hundred and 9 Tens. We *write* the 9 Tens in the Product, and *retain* the *1 Hundred* in the mind. Finally, multiplying 9 Hundreds by 3, we have 27 Hundreds for a Partial Product, to which we add the 1 Hundred retained, and *write* the Sum, *28 Hundreds*, in the Product. Our final Product is 2,892; the same as in the first Solution.

FIRST SOLUTION.

$$\begin{array}{r} 964 \\ 3 \\ \hline 12 \\ 18 \\ 27 \\ \hline 2,892 \end{array}$$

SECOND SOLUTION.

$$\begin{array}{r} 964 \\ 3 \\ \hline 2,892 \end{array}$$

These 2 methods of Multiplication differ in this only; in the *first* the *Partial Products are written*, while in the *second only the final Product is written.* The written work in the second is much *shorter* than in the first.

DEFINITIONS.

1. ***Long Multiplication*** is the method of *Multiplication* used where we obtain the final Product by *writing out the Partial Products* and finding their Sum.

2. ***Short Multiplication*** is the method of *Multiplication* used where, after writing the Multiplicand and Multiplier, we *write the final Product only, performing the rest of the work mentally.*

Find the Products by Short Multiplication in these

EXERCISES FOR THE SLATE AND BOARD.

879,567 × 2	543,763 × 3	825,046 × 4
342,354 × 5	243,652 × 6	105,824 × 9
243,054 × 6	540,324 × 8	134,564 × 9
536,726 × 6	947,540 × 8	302,132 × 9

LESSON LXXXVIII.

EXAMPLE B. Divide 2,892 by 3.

EXPLANATION. 1st. The first Solution is full.

2d. In the second Solution we shorten the work. Finding that 28 Hundreds divided by 3 give 9 Hundreds, we write 9 Hundreds in the Quotient. We then multiply 9 Hundreds by 3; and, instead of *writing* the Product, 27 Hundreds, we *retain it in the mind,* and *subtracting* it from 28 Hundreds *mentally,* find 1 Hundred remaining. Next, we *mentally* unite the 9 Tens with the 1 Hundred left, and retained in the mind, and have 1 Hundred and 9 Tens, or 19 Tens, for a new Partial Dividend, which we retain in the mind. Finding that 19 Tens divided by 3 give 6 Tens, we write 6 Tens in the Quotient. Multiplying 6 Tens by 3 and obtaining 18 Tens for a Product, we *subtract* 18 Tens from 19 Tens *men-*

FIRST SOLUTION.

```
  964 Quotient.
    3 Divisor.
-----
2,892 Dividend.
27
---
 19
 18
 ---
  12
  12
  ---
```

SECOND SOLUTION.

```
  964
    3
-----
2,892
```

tally; and, finding 1 Ten remaining, we *retain* this *in the mind.* Finally, uniting the 2 Ones of the Dividend with the 1 Ten retained, we have for our last Partial Dividend 1 Ten and 2 Ones, or 12. We *do not write this;* but, *dividing* by 3 *mentally,* we write the result, 4, in the Quotient. We have 964 for our final Quotient.

The method used in the *second Solution* is much *shorter* than that in the first, since in the second *we perform the work mentally, without writing the Partial Dividends and Products.*

DEFINITIONS.

1. ***Long Division*** is the method of Division used where we obtain the Quotient by *writing the Partial Dividends and Products.*

2. ***Short Division*** is the method of Division used where we obtain the Quotient *without writing the Partial Dividends and Products.*

Perform the work by Short Division in the following

Exercises for the Slate and Board.

I.

789,156 ÷ 2	591,732 ÷ 2	769,518 ÷ 2
768,927 ÷ 3	857,421 ÷ 3	257,613 ÷ 3
736,928 ÷ 4	175,628 ÷ 4	279,536 ÷ 4
623,715 ÷ 5	859,235 ÷ 5	973,265 ÷ 5
157,326 ÷ 6	312,714 ÷ 6	517,896 ÷ 6
548,765 ÷ 7	227,353 ÷ 7	257,327 ÷ 7
264,976 ÷ 8	512,760 ÷ 8	512,064 ÷ 8
325,719 ÷ 9	289,881 ÷ 9	123,453 ÷ 9

II.

Divide 7,206,480 by 2;	by 3;	by 4;	by 5;	by 6.
Divide 14,405,760 by 2;	by 3;	by 4;	by 5;	by 6.
Divide 5,040,720 by 2;	by 3;	by 4;	by 5;	by 6.
Divide 833,280 by 4;	by 5;	by 6;	by 7;	by 8.

LESSON LXXXIX.

MULTIPLICATION TABLE.					*DIVISION TABLES.*							
10	*Times*	1 is 10,	**10**	*are in*	10	1 Time,	1	*is in*	10	**10** Times,
10		2 are 20,	10		20	2 Times,	2		20	10 Times,
10		3 are 30,	10		30	3 Times,	3		30	10 Times,
10		4 are 40,	10		40	4 Times,	4		40	10 Times,
10		5 are 50,	10		50	5 Times,	5	*are in*	50	10 Times,
10		6 are 60,	10		60	6 Times,	6		60	10 Times,
10		7 are 70,	10		70	7 Times,	7		70	10 Times,
10		8 are 80,	10		80	8 Times,	8		80	10 Times,
10		9 are 90,	10		90	9 Times,	9		90	10 Times,
10		10 are 100.	10		100	10 Times.	10		100	10 Times.

	I.			II.		
Multiplicands.	10	10	10	7	8	9
Multipliers.	7	8	9	10	10	10
Products.	70	80	90	70	80	90

In these two sets of Examples, *the numbers multiplied together are the same,* and hence the *Products must be the same* in both sets.

From the Principle on page 93, we see that *if two numbers are to be multiplied together either may be used as the Multiplicand, and the other as the Multiplier.* Hence we use the Multiplicands of the first set as Multipliers in the second set.

The *Products are the same in both sets; and are formed in the second set by annexing a 0 to each Multiplicand.*

EXAMPLE. Multiply 25 by 10.

SOLUTION.

25	*Multiplicand.*
10	*Multiplier.*
250	*Product.*

EXPLANATION. Writing the 10 under the 25 so that the 1 shall stand under the 5, we bring down

the 25 into the Product, and then *annex a cipher,* to obtain the final Product. By annexing the cipher the 5 Ones are made 5 Tens, which are 10 times as many as 5 Ones; and the 2 Tens are made 2 Hundreds, which are 10 times as many as 2 Tens. Therefore, since 250 is 10 times as large as 25, it is the true Product. Hence we have this

INFERENCE.

Annexing a cipher at the right of a number multiplies it by 10.

Multiply each of the following numbers by 10:

125; 896; 2,570; 37,896; 543,768; 5,020; 500.

LESSON XC.

FACTORS AND PRODUCT.

In each of the first 4 Equations, at the right, 2 numbers are multiplied together to make a third number.

FACTORS.	PRODUCTS.
2×3	$= 6$
2×5	$= 10$
3×3	$= 9$
3×5	$= 15$
$2 \times 3 \times 5$	$= 30$
$2 \times 3 \times 7$	$= 42$

The *numbers* thus *multiplied together to make a Product are named* ***Factors.*** *Factor* means *maker;* and *Product* means *something made,* or *produced.*

Thus, 2 and 5 are the *Factors* of 10, their Product; and 2, 3 and 5 are the *Factors* of 30, their Product.

The numbers 6, 10, 9, 15, 30, and 42, given above, are *composed* of the numbers multiplied together to make them, and are therefore named ***Composite Numbers.***

The numbers multiplied together to form another number are named its ***Component Factors.***

DEFINITIONS.

1. A ***Factor*** is one of the numbers multiplied together to produce another number.

2. A ***Prime Factor*** is any Factor which cannot itself be produced by multiplying together *other* Factors.

3. A ***Prime Number*** is any number which cannot be produced by multiplying together *other* numbers.

4. A ***Composite Number*** is any number which can be produced by multiplying together *other* numbers.

5. A ***Component Factor*** is one of the Factors multiplied together to produce a Composite Number.

6. An ***Even Number*** is any number having 2 as one of its Prime Factors.

7. An ***Odd Number*** is any number not having 2 as one of its Prime Factors.

Remarks.

1. Every number whose right-hand figure is *zero,* or is exactly divisible by 2, is an Even Number.

2. Every Even Number, except 2, is Composite.

3. Every number whose right-hand figure is an Odd Number is itself an Odd Number.

Of the following numbers what ones are Prime, and what ones Composite? What Even, and what Odd?

2,	4,	9,	11,	8,	16,	21,	25,	17,	63,	164.
5,	7,	6,	10,	13,	14,	15,	19,	22,	27,	120.

Exercises for the Slate and Board.

Multiplication.

Multiply 7,859,634 by 5; by 6; by 7; by 8; by 9.
Multiply 9,473,857 by 4; by 6; by 7; by 8; by 9.
Multiply 5,837,918 by 3; by 5; by 7; by 8; by 9.

Division.

Divide 6,053,040 by 5; by 6; by 7; by 8; by 9.
Divide 6,562,584 by 3; by 4; by 7; by 8; by 9.

LESSON XCI.

We have heretofore seen that any Product can be divided by either of the numbers multiplied together to produce it. In the same manner any Product formed of any number of Factors can be divided by any of its Factors, or by all of them in succession.

We will take the Prime Factors 13, 7, and 3, and find their Product, and then divide this Product by its Prime Factors, 13, 7, and 3, and see what we shall have for a Quotient.

1.—*Multiplication.*			2.—*Division.*
1st Factor,	**13**	**13**	*2d Quotient.*
2d Factor,	**7**	**7**	*2d Divisor.*
	91	91	*1st Quotient.*
3d Factor,	**3**	**3**	*1st Divisor.*
Product,	273	273	*1st Dividend.*

1st, MULTIPLICATION. Multiplying ***13*** and ***7*** together, and then multiplying their Product, 91, by ***3,*** the third Factor, we obtain 273 for a final Product.

2d, DIVISION. *Dividing this Product,* 273, by ***3,*** the last *Multiplier,* we obtain *91* for our *first Quotient.*

Dividing 91 *by* ***7,*** *another of the 3 Factors multiplied together,* we obtain, for a *Quotient,* ***13,*** *the other of the 3 Factors.*

If, now, we should divide our last Quotient, 13, by ***13,*** the final Quotient would be *1.*

We have divided the number 273 by each of its Prime Factors in succession, and obtained *1* for a final Quotient. Thus it is evident that, in whatever order we multiply the numbers together, we may divide the Pro-

duct by all the numbers in succession, in a *contrary order.*

Hence we have the following

rinciple in Division. A.

Any Composite Number can be divided by any one of its Component Prime Factors, or *by all of them in succession, using each Quotient for a new Dividend.*

Since any given *Composite Number can be formed by multiplying together its Prime Factors,* and *cannot be formed by multiplying together any other Prime Factors,* it is evident that it *cannot be exactly divided by any other Prime Factor.* Hence

Principle in Division. B.

No Composite Number can be exactly divided by any Prime Number which is not one of its Prime Factors. Hence,

To find the Prime Factors of any Number:

RULE.

Divide the Number by any Prime Number that will exactly divide it; then divide the Quotient by any Prime Number that will exactly divide it; and so continue to divide until a Quotient is obtained that is itself a Prime Number. The several Divisors and the final Quotient will be the Prime Factors of the given Prime Number.

Find and write all the Prime Factors of each of the following Composite Numbers:

16	24	30	40	50	60	75	85	100
18	25	32	42	54	64	80	90	112
20	27	35	45	55	70	81	95	120
21	28	36	48	56	72	84	96	128

LESSON XCII.

MULTIPLYING BY A COMPOSITE NUMBER.

Example. Multiply 365 by 18.

Explanation. 1st. We separate the Multiplier, 18, into the Component Factors 2 and 9.

2d. We multiply 365 by 9, one of the Component Factors of 18, thus taking 365 *9 times.*

3d. We multiply this Product, 3,285, by 2, the other Component Factor of 18; thus taking 9 times 365 *2 times.* Since 2 times 9 times are 18 times, we have thus taken 365 18 times; or multiplied 365 by 18.

SOLUTION.

365	*Multiplicand.*
9	*1st Multiplier.*
3,285	
2	*2d Multiplier.*
6,570	*Product.*

Instead of 2 and 9, as the Component Factors of 18, we might have taken 3 and 6, or 2, 3 and 3. Multiplying by these, we should have obtained the same Product. Hence,

To Multiply by any Composite Number:

Rule.

I. *Separate the Multiplier into any number of Component Factors.*

II. *Multiply the Multiplicand by one of these Factors; then this Product by another, and so on till all the Component Factors have been used as Multipliers. The final Product will be the true Product.*

By using Component Factors as Multipliers,

Multiply 23,546 by 12;	by 14;	by 15;	by 18.
Multiply 37,985 by 24;	by 27;	by 35;	by 36.
Multiply 85,896 by 42;	by 45;	by 48;	by 49.
Multiply 98,583 by 56;	by 63;	by 64;	by 70.
Multiply 56,874 by 75;	by 81;	by 84;	by 96.

LESSON XCIII.

DIVIDING BY A COMPOSITE NUMBER.

1.—*Multiplication.*		2.—*Division.*	
Multiplicand.	376	376	*Final Quotient.*
1st Multiplier.	7	7	*2d Divisor.*
1st Product.	2,632	2,632	*1st Quotient.*
2d Multiplier.	2	2	*1st Divisor.*
Final Product.	5,264	5,264	*Dividend.*

Above we have multiplied 376 by 14, by using 7 and 2, the Component Factors of 14, as Multipliers. The Product is 5,264. If, now, this Product be divided by 14, our Multiplier, the Quotient will be the Multiplicand, 376. But, above, we have obtained this result by dividing by 2 and 7, the Component Factors of 14.

We notice that in our Division we retraced all our work of Multiplication. Hence,

To Divide by any Composite Number:

Rule.

I. *Separate the Divisor into any number of Component Factors.*

II. *Divide the Dividend by any one of these Factors; then divide the Quotient by another Factor, and so on till all the Factors have been used as Divisors.*

The final Quotient will be the true Quotient.

By using Component Factors as Divisors,

Divide	40,320 by 12;	by 14;	by 15;	by 16.
Divide	816,480 by 24;	by 27;	by 35;	by 36.
Divide	4,445,280 by 42;	by 45;	by 48;	by 49.

The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, always *express Number*. They are therefore called ***Numeral Figures,*** or ***Numerals.*** Zero (0) *never itself expresses Number*, but shows the *absence of* Number.

LESSON XCIV.

MULTIPLYING BY ANY NUMBER CONSISTING OF 1 WITH CIPHERS ANNEXED.

EXAMPLE. Multiply 365 by 100.

EXPLANATION. 10 and 10 may be regarded as the Component Factors of 100. Hence we can multiply by 100 by multiplying by 10 and 10. But we may multiply by 10 by annexing *one cipher* at the right of the Multiplicand. Hence, to multiply by 10 *twice*, that is, by 100, we must annex *two ciphers*. Therefore, $365 \times 100 = 36{,}500$. The ciphers at the right of 1, in 100, or 1,000, or 10,000, or any other number, show how many times 10 is a factor in the number. Hence,

TO MULTIPLY BY ANY NUMBER CONSISTING OF 1 WITH CIPHERS ANNEXED AT THE RIGHT:

RULE.

Annex at the right of the Multiplicand as many ciphers as there are in the Multiplier. The result will be the true Product.

Multiply 785 by 1,000; by 10,000; by 100,000.

MULTIPLYING BY ANY NUMBER CONSISTING OF ONE NUMERAL FIGURE WITH CIPHERS ANNEXED:

The Component Factors of 500 may be taken as 5 and 100; since these numbers, multiplied together, make 500. Hence, we can multiply by 500 by multiplying by its Factors, 5 and 100. It is also plain that we can multiply by 5,000 by multiplying by its Factors, 5 and 1,000. Therefore, to multiply by 5,000, we can first multiply by 5, and then multiply the Product thus obtained by 1,000, by annexing three ciphers. Hence,

TO MULTIPLY BY A NUMBER CONSISTING OF A SINGLE NUMERAL FIGURE, WITH CIPHERS ANNEXED:

RULE.

I. *Multiply the Multiplicand by the left-hand figure of the Multiplier.*

II. *Annex at the right of this Product as many Ciphers as there are standing at the right in the Multiplier. The result will be the true Product.*

EXERCISES FOR THE SLATE AND BOARD.

7,854 × 70	3,586 × 700	734 × 5,000
9,873 × 90	8,962 × 800	548 × 7,000
5,420 × 80	5,423 × 900	820 × 9,000

In the number 555, the figures do not each express the same value. The 5 at the right stands for 5 Ones; the 5 at the left of this for 5 Tens, or 50; and the 5 at the left for 5 Hundreds, or 500. Hence, the value of each figure 5 is determined by its place or *locality.*

DEFINITIONS.

1. The ***Simple Value*** of any Numeral Figure is the value which it expresses when standing in the *place of Ones.*

2. The ***Local Value*** of any Numeral Figure is the value which it expresses when standing in *any place.*

REMARKS.

1. The *Simple Value* of a Numeral Figure is *always* the *same.*

2. The *Local Value* of a Numeral Figure *changes as often as* the *place* of the figure is *changed.*

3. When a Numeral Figure *stands in the place of Ones,* its *Simple and Local Value are the same.*

4. The figure 0 has *no value,* either Simple or Local.

LESSON XCIV.

EXAMPLE A. Multiply 549 by 375.

SOLUTION.

549	*Multiplicand.*
375	*Multiplier.*
2,745	= *5 Times 549.*
38,430	= *70 Times 549.*
164,700	= *300 Times 549.*
205,875	= *375 Times 549.*

EXPLANATION. 1st. We multiply 549 by 5, or take it 5 times, and write the Partial Product, 2,745. 2d. We multiply 549 by 70, or take it 70 times, and write the Partial Product, 38,430. 3d. We multiply 549 by 300, or take it 300 times, and write the Partial Product, 164,700. 4th. We add the 3 Partial Products, and take their Sum as the final Product.

Thus we have taken 549, our Multiplicand, *300 times*, and *70 times*, and *5 times*, or *375 times*. Hence, we have the true Product.

EXAMPLE B. Multiply 249 by 305.

SOLUTION.

249
305
1,245
74,700
75,945

EXPLANATION. We first take 249 *5 times*, then *300 times*, and, writing the Partial Products, add them.

We do not have any Partial Product arising from multiplying by *0* in the Multiplier; since taking 249 *no* (0) *times is not taking it at all.* The other Partial Products are written as in the preceding Solution.

For each Numeral Figure of the Multiplier there is a corresponding Partial Product, obtained by multiplying the whole Multiplicand by this Figure. Hence,

TO MULTIPLY ONE NUMBER BY ANOTHER:

RULE.

I. *Write the Multiplier under the Multiplicand.*

II. *Commencing at the right, multiply the whole Multiplicand by each Numeral Figure of the Multiplier, regarding the Local Value of each Figure, and write the Partial Products.*

III. *Add the Partial Products, and take their Sum as the final Product.*

Exercises for the Slate and Board.

Multiply 3,582 by 125;	by 342;	by 976;	by 748.
Multiply 7,643 by 502;	by 430;	by 900;	by 708.
Multiply 9,536 by 739;	by 608;	by 968;	by 374.

Multiply 5,894 by 5,423;	by 4,672;	by 6,702;	by 3,064.
Multiply 3,698 by 5,008;	by 7,041;	by 8,302;	by 7,006.

Multiply 7,874 by 25,376;	by 30,708;	by 50,023.
Multiply 657 by 46,002;	by 50,007;	by 10,101.

LESSON XCV.

EXAMPLE. Multiply 347 by 235.

EXPLANATION. The first of these two Solutions is in the form heretofore used.

The second Solution differs from the first only in having ciphers omitted at the right of the Partial Products.

1ST SOLUTION.	2D SOLUTION.
347	347
235	235
1,735	1,735
10,410	10,41
69,400	69,4
81,545	81,545

We observe that the second Partial Product, 10,41, is found by multiplying 347 by 3; and the third, 69,4, by multiplying 347 by 2. But, since the figure 3 in the Multiplier stood for 30, or 3 × 10, 10,41 must still be

multiplied by 10. It has been written *one place to the left,* so as to leave room for a *cipher at the right,* on multiplying it by 10. So, also, the Partial Product 69,4 has been written *two places to the left,* so as to leave room for *two ciphers at the right,* on multiplying by 100.

In the second Solution the *right-hand figure* of each Partial Product is written *directly below the corresponding figure of the Multiplier.* Hence, when the ciphers at the right of the Partial Products are omitted,

TO WRITE THE PARTIAL PRODUCTS:

RULE.

Multiply the Multiplicand by each Numeral Figure of the Multiplier, and write the Partial Products so that the RIGHT-HAND FIGURE *in each shall stand* DIRECTLY BELOW THE CORRESPONDING FIGURE OF THE MULTIPLIER.

If there are any ciphers at the right of the first Numeral Figure of the Multiplier, write the same number of ciphers at the right of the first Partial Product.

The second Partial Product, 10,41, is read 10,410; and the third, 69,4, is read 69,400. Hence,

TO READ THE PARTIAL PRODUCTS:

RULE.

Read each Partial Product THE SAME AS IF THE OMITTED CIPHERS WERE WRITTEN.

EXERCISES FOR THE SLATE AND BOARD.

Multiply 6,852 by 237;	by 543;	by 678;	by 906.
Multiply 3,826 by 460;	by 307;	by 500;	by 1,060.
Multiply 6,978 by 1,502;	by 2,007;	by 2,030;	by 9,706.
Multiply 8,059 by 6,025;	by 3,006;	by 9,408;	by 3,500.
Multiply 7,684 by 2,643;	by 5,489;	by 3,762;	by 7,563.

LESSON XCVI.

EXAMPLE. Find the Product arising from multiplying together 468 and 12.

FIRST SOLUTION.		SECOND SOLUTION.	
12	*Multiplicand.*	468	*Multiplicand.*
468	*Multiplier.*	12	*Multiplier.*
96	= *8 times 12.*	96	= *12 times 8.*
72	= *60 times 12.*	72	= *12 times 60.*
4,8	= *400 times 12.*	4,8	= *12 times 400.*
5,616	= *468 times 12.*	5,616	= *12 times 468.*

EXPLANATION. As the Product will be the same whichever of the two numbers be taken as Multiplier, in the first Solution we have made 468 Multiplier, and in the second Solution 12. The first Solution is in the usual form.

In the second Solution we first multiplied 8 by 12; then 6, or 60, by 12; and finally 4, or 400, by 12. Thus we have taken 400, and 60, and 8, 12 times; or have taken 468 12 times. The *Partial Products stand in the same order*, and *are the same, in both Solutions.*

In the second Solution, we find it *easier* to obtain the Product of 8 multiplied by 12 by multiplying *12 by 8*, the Product being the same in both cases. So, also, to find the Product of 60 multiplied by 12, we multiply 12 by 60. In the same manner, we find the Product of 400 × 12 by multiplying 12 by 400.

The peculiarity in the second Solution consists in *multiplying first the right-hand figure of the Multiplicand by the entire Multiplier, then the next figure of the Multiplicand by the entire Multiplier*, and so on until all the figures of the Multiplicand have been multiplied by the Multiplier. But, though we consider 12 the

Multiplier, we obtain the Partial Products by using the figures of the Multiplicand as Multipliers, and 12 as Multiplicand.

By the method used in the second Solution, obtain the Products in the following

EXERCISES FOR THE SLATE AND BOARD.

Multiply 473 by 13; by 25; by 36; by 47; by 58.
Multiply 568 by 45; by 73; by 84; by 39; by 96.

Multiply 2,437 by 123; by 234; by 543; by 736.
Multiply 5,283 by 542; by 745; by 827; by 976.

LESSON XCVII.

EXAMPLE A. Multiply 236 by 12.
EXAMPLE B. Divide 2,832 by 12.

SOLUTION OF EX. A.

Multiplicand.	236
Multiplier.	12
12 times 6 =	72
12 times 30 =	36
12 times 200 =	2,4
	2,832

SOLUTION OF EX. B.

236	*Quotient.*
12	*Divisor.*
2,832	*Dividend.*
2,4	*= 12 times 200.*
43	
36	*= 12 times 30.*
72	
72	*= 12 times 6.*

EXPLANATION. In the Solution of Example A, the Product is obtained by the method used in the last Lesson. We find the Product to be 2,832; the same as the Dividend in Example B. Our Multiplicand is 236, our Multiplier 12, and our Product 2,832.

In Example B we are required to divide this Product, 2,832, by the Multiplier, 12. Hence, according to Prin-

ciple 5 in Division, on page 106, our Quotient must be the same as our Multiplicand, 236. We will obtain it by the Solution.

Writing the Divisor above the Dividend, with a line between them, we see that we are to find *a Multiplicand which when multiplied by 12 will give 2,832* for a Product.

1st. We first ask: What is the greatest number of Hundreds which, when written in the Multiplicand (or Quotient) and multiplied by 12, will give a Partial Product not exceeding 28 Hundreds? Finding this number of Hundreds to be 2, we write 2 Hundreds in the Multiplicand (or Quotient). Multiplying 2 Hundreds by 12, or 12 by 200, and subtracting the Partial Product, 24 Hundreds, from 28 Hundreds, 4 Hundreds are left.

2d. Bringing down the 3 Tens from 2,832, and writing them at the right of 4 Hundreds, we have 4 Hundreds and 3 Tens, or 43 Tens, for our next Partial Dividend. We now ask: What is the greatest number of Tens which, when written in the Multiplicand and multiplied by 12, will give a Partial Product not exceeding 43 Tens. Finding this number of Tens to be 3, we write 3 Tens in the Multiplicand (or Quotient), and multiplying the 3 Tens by 12 (or 12 by 30), and subtracting the Partial Product, 36 Tens, from 43 Tens, we write the Remainder, 7 Tens.

3d. Finally, bringing down the 2 Ones from 2,832, and writing them at the right of our 7 Tens, we have 7 Tens and 2 Ones, or 72 Ones, for a Partial Dividend. Finding that 6 Ones, when written in the Multiplicand (or Quotient) and multiplied by 12, will give 72 Ones, we write 6 Ones as the last figure in the Multiplicand (or Quotient). Multiplying and subtracting, as before,

we have *no Remainder.* Therefore, our Multiplicand, or Quotient, is 236. Hence,

To Divide one Number by another:

Rule.

I. *Write the Divisor above the Dividend, separating them by a horizontal line.*

II. *Commencing at the left, take as a Partial Dividend such a part of the entire Dividend as, without regarding its Local Value, will contain the Divisor* AT LEAST ONCE *and* NOT MORE THAN NINE TIMES; *and, determining the first figure of the Quotient, write it in its place, giving it its proper Local Value.*

III. *Multiply this Divisor by the Quotient figure, and subtract the result from the Partial Dividend.*

IV. *Write the next figure of the Dividend at the right of the Remainder; and, using the number thus formed as a new Partial Dividend, determine the second figure of the Quotient in the same manner as the first was obtained; and multiply and subtract as before.*

Proceed in this manner till all the figures of the Quotient are obtained, and the work is completed.

Remarks.

1. If any *Partial Product* is *greater* than the *Partial Dividend* under which it is written, the corresponding *Quotient figure* is *too large,* and *must be diminished.*

2. If any *Remainder* is *greater* than the *Divisor,* the corresponding *Quotient figure is too small,* and *must be increased.*

3. Whenever any *Partial Dividend* is *less* than the *Divisor,* the corresponding *Quotient figure* is *0;* and the next *Partial Dividend is formed directly from this,* by writing the next figure of the Dividend at the right of it.

EXERCISES FOR THE SLATE AND BOARD.

Divide 32,760	by 12;	by 13;	by 14;	by 15.
Divide 1,970,640	by 23;	by 34;	by 45;	by 56.
Divide 375,480	by 149;	by 298;	by 447;	by 745.
Divide 124,488	by 247;	by 494;	by 741;	by 1,729.

LESSON XCVIII.

EXAMPLE. Divide 1,589 by 58.

EXPLANATION. We divide as heretofore, and obtain 27 for a Quotient; but, on subtracting the *last* Partial Product, we have 23 for a *Remainder*. This *last Remainder* is named the ***Final Remainder***. It is never called the Difference, as in Subtraction.

SOLUTION.

```
   27 Quotient.
   58 Divisor.
 ----
1,589 Dividend.
1,16
 ----
  429
  406
 ----
   23 Remainder.
```

DEFINITION.

The ***Final Remainder*** in Division is that *part of the Dividend left,* still *undivided,* after obtaining all the figures of the Quotient.

EXERCISES FOR THE SLATE AND BOARD.

Divide 23,578	by 35;	by 59;	by 68;	by 79.
Divide 376,982	by 143;	by 256;	by 374;	by 578.
Divide 438,796	by 527;	by 743;	by 963;	by 829.
Divide 25,762,159	by 159;	by 1,524;	by 3,284.	
Divide 57,349,284	by 6,023;	by 7,009;	by 8,219.	
Divide 29,513,784	by 374;	by 183;	by 921;	by 548.
Divide 73,182,546	by 1,316;	by 589;	by 918;	by 3,600.
Divide 10,020,010	by 101;	by 1,010;	by 10;	by 100.
Divide 10,000,000	by 10;	by 100;	by 1,000;	by 10,000.

LESSON XCIX.

We multiply by 10, 100, 1,000, &c., by annexing at the right of the Multiplicand as many ciphers as stand at the right in the Multiplier.

According to Principle 5, on page 106, if the Product be divided by the Multiplier the Quotient will be the Multiplicand. But it is evident that we can obtain the Multiplicand, or Quotient, in such cases, by dropping at the right of the Product, or Dividend, the same ciphers which we have just annexed. Hence,

TO DIVIDE BY 10, 100, 1,000, &C., WHEN THE DIVIDEND HAS AS MANY CIPHERS AT THE RIGHT AS ARE FOUND IN THE DIVISOR:

RULE.

Drop at the right of the Dividend as many ciphers as stand at the right in the Divisor. The figures remaining will be the Quotient.

EXERCISES FOR THE SLATE AND BOARD.

Divide 57,830,000 by 100;	by 1,000;	by 10,000.
Divide 21,700,000 by 10;	by 10,000;	by 100,000.
Divide 65,430,000 by 10;	by 1,000;	by 10,000.

EXAMPLE 1. Divide 54,768 by 100.

SOLUTION.

54,700 + 68.
54,700 ÷ 100 = 547 *Quotient.*
Final Rem., 68.

EXPLANATION 1. Before dividing, we separate our Dividend into *two parts*, 54,700 and 68. Next we divide the first part, 54,700 by 100, by dropping two ciphers, and have 547 for a Quotient. It is plain that 68 will not contain 100 even once. Hence it will be our Final Remainder. Our Quotient is 547, and our Final Remainder 68.

EXAMPLE 2. Divide 54,768 by 1,000.

SOLUTION.

54,768 = 54,000 + 768
54,000 ÷ 1,000 = 54 *Quo't.*
Final Remainder, 768.

EXPLANATION. Removing the three right-hand figures, 768, and supplying their places with ciphers, we separate the Dividend into two parts, 54,000 and 758. Dividing 54,000 by 1,000, by rejecting the three ciphers at the right, we obtain 54 for our Quotient. The other part of the Dividend, 768, being less than the Divisor, will not contain it even once, and hence will be our Final Remainder.

On examination, we find that when we divide by 100, 1,000, &c., the figures removed at the right of the Dividend form our Final Remainder, and the figures at the left of these, in the Dividend, form our Quotient. Hence,

TO DIVIDE BY 10, 100, 1,000, &C.:

RULE.

Remove at the right of the Dividend as many figures as there are ciphers in the Divisor, and take the number composed of the figures so removed for the Final Remainder, and the number composed of the figures at the left of these for the Quotient.

EXAMPLE 3. Divide 7,958 by 400.

1ST SOLUTION.		2D SOLUTION.	
19	*Quo't.*	19	*Quo't.*
400	*Div'r.*	4 ¦ 00	*Div'r.*
7958	*Divid'd.*	79 ¦ 58	*Divid'd.*
400		4	
3958		39	
3600		36	
358	*Fin. Rem.*	358	*Fin. Rem.*

EXPLANATION. The first Solution is in the form heretofore given. We see that the ciphers in the Divisor appear in the Partial Products, but do not affect the work so as to change either the Quotient or the Final Remainder.

The figures 58, at the right in the Dividend, appear in the Final Remainder unchanged. Hence, in the second Solution the work is shortened, by cutting off the ciphers at the right in the Divisor, and the figures 58 at the right in the Dividend, and then annexing the 58 to the second Remainder. Therefore,

To Divide by a Number with Ciphers at the Right:

Rule.

I. *Cut off the ciphers at the right of the Divisor, and an equal number of figures at the right of the Dividend.*

II. *Divide the Dividend, thus changed, by the changed Divisor, and use the Quotient thus obtained as the true Quotient.*

III. *Annex to the last Remainder the figures cut off from the Dividend, and use the number thus formed as the Final Remainder.*

Exercises for the Slate and Board.

Divide 78,546 by 10;	by 100;	by 1,000.
Divide 57,968 by 100;	by 1,000;	by 10,000.
Divide 30,102 by 10;	by 100;	by 1,000.
Divide 73,001 by 100;	by 1,000;	by 10,000.
Divide 237,849 by 60;	by 700;	by 8,000.
Divide 546,789 by 80;	by 900;	by 70,000.
Divide 107,050 by 90;	by 500;	by 8,000.
Divide 700,520 by 30;	by 650;	by 3,500.
Divide 672,518 by 230;	by 3,100;	by 30,100.
Divide 127,950 by 100;	by 7,200;	by 50,001.
Divide 718,312 by 101;	by 2,001;	by 10,001.

LESSON C.

EXAMPLE. Multiply 15 by 14.

	1st Solution.	2d Solution.
Multiplicand.	15 =	3×5
Multiplier.	14 =	2×7
	60	$3 \times 5 \times 2 \times 7$
	15	2
Product.	210	14
		5
		70
		3
		210 *Prod't.*

EXPLANATION. 1st. The first Solution is in the usual form, and the Product is 210. 2d. In the second Solution, we factor 15 into 3 and 5, and 14 into 2 and 7. Writing the Factors of both Multiplicand and Multiplier in a line, with the Sign × between, for the Factors of the Product, we multiply them together, and obtain 210 for a Product. Hence,

General Principle in Multiplication.

The Product is composed of the Factors of the Multiplicand and Multiplier, and NO OTHERS.

Since the *Product* in Multiplication *becomes* the *Dividend* in Division, the *Multiplier* the *Divisor*, and the *Multiplicand* the *Quotient*, therefore, in Division, *when there is no Remainder*, we have

General Principles in Division.

1. *The Dividend is composed of the Factors of the Divisor and Quotient, and* NO OTHERS.

2. *If the Factors of the Divisor be rejected from the Dividend, the remaining Factors will be those of the Quotient.* Hence,

TO MULTIPLY BY ANY NUMBER:

RULE.

Connect the Factors of the Multiplier with those of the Multiplicand by the Sign ×. The Product of these Factors will be the true Product.

Hence, also, *when there is no Remainder,*

TO DIVIDE BY ANY NUMBER:

RULE.

From the Dividend remove Factors equal to those of the Divisor. The Product of the remaining Factors will be the true Quotient.

We saw, on page 98, that the Sign of Division, $\div$, is made from the Sign Minus, $-$.

We may use the Sign Minus in still another manner to show that one number is to be *divided by* another. We may show that 12 are to be *divided by* 3, in the manner seen at the right.

$$\begin{array}{l} \textit{Dividend.} \\ \textit{Divisor.} \end{array} \frac{12}{3} = 4 \textit{ Quotient.}$$

1st: We write the Dividend, 12. 2d: We write the Sign Minus below the Dividend. 3d: We write the Divisor below the Sign Minus.

If we make this Expression the First Member of an Equation, the Second Member will be the Quotient. The Expression $\frac{12}{3} = 4$ is read: "*12 divided by 3 equal 4.*" The Quotient, 4, is called the ***Value*** of the Expression $\frac{12}{3}$.

Find the Value of each Expression in the following

EXERCISES FOR THE SLATE AND BOARD.

I.

$\frac{15}{5} = ?$ $\frac{27}{3} = ?$ $\frac{42}{7} = ?$ $\frac{72}{8} = ?$ $\frac{63}{7} = ?$

II.

$\frac{936}{3}$ $\frac{675}{5}$ $\frac{959}{7}$ $\frac{1792}{8}$ $\frac{3753}{9}$

EXAMPLE. Divide 378 by 42.

EXPLANATION. From the second General Principle in Division we see that the Quotient consists of those Factors of the Dividend which are not in the Divisor. Hence, by the method shown on page 129 we factor 378 into $2 \times 3 \times 7 \times 9$, and 42 into $2 \times 3 \times 7$. Writing the Factors of the Dividend above those of the Divisor, for Division, we reject from the Dividend the Factors $2 \times 3 \times 7$, of the Divisor, as directed by the preceding Rule, and have the remaining Factor, 9, for our Quotient.

SOLUTION.

$378 = 2 \times 3 \times 7 \times 9.$

$42 = 2 \times 3 \times 7.$ Hence,

$$\frac{378}{42} = \frac{2 \times 3 \times 7 \times 9}{2 \times 3 \times 7} = 9 \textit{ Quotient.}$$

Obtain the Quotients by factoring in the following

EXERCISES FOR THE SLATE AND BOARD.

$\frac{132}{66}$ $\frac{195}{39}$ $\frac{2205}{441}$ $\frac{2940}{420}$ $\frac{3888}{432}$ $\frac{8640}{1728}$

LESSON CI.

Since our Quotient always consists of those Factors of the Dividend remaining after taking away the Factors of the Divisor, it is plain that (Principle 3) putting any *new* Factor into the *Dividend* does in effect put that Factor into the *Quotient*; and that (Principle 4) removing from the *Dividend* any Factor already there in effect removes it from the *Quotient*.

It is also evident that (Principle 5) if any *new* Factor be put into the *Divisor*, when we take the Factors of the

Divisor from the Dividend we must also take this *new* Factor from the *Dividend,* and thus in effect take it from the *Quotient,* or divide the Quotient by it; and that (Principle 6) if any one of the Factors of the Divisor be removed from the Divisor before taking the Factors of the Divisor from the Dividend, the Factor so removed will not be taken from the Dividend, as it should be, but will leave the corresponding Factor of the Dividend in the Quotient, and will thus, in effect, multiply the Quotient by this Factor.

It is also clear that (Principle 7) if any new Factor be put into both Dividend and Divisor, the one in the Divisor will cause the removal of the one in the Dividend, and hence the Quotient will not be affected. And (Principle 8) if the same Factor be removed from both the Dividend and Divisor, the final effect will be the same as if it had remained in both till all the Factors of the Divisor were taken from the Dividend. Hence the Quotient will not be affected.

Therefore, we shall have as

General Principles of Division:

3. *Multiplying the Dividend by any Factor in effect multiplies the Quotient by that Factor.*

4. *Dividing the Dividend by any Factor in effect divides the Quotient by that Factor.*

5. *Multiplying the Divisor by any Factor in effect divides the Quotient by that Factor.*

6. *Dividing the Divisor by any Factor in effect multiplies the Quotient by that Factor.*

7. *Multiplying both Dividend and Divisor by the same Factor does not affect the Quotient.*

8. *Dividing both Dividend and Divisor by the same Factor does not affect the Quotient.*

Dividing both Dividend and Divisor by the same Factor is the same as *rejecting* that Factor from both.

DEFINITION.

Cancellation is rejecting equal Factors from both Dividend and Divisor.

EXAMPLE. Divide 60 by 15.

SOLUTION.

$$\begin{matrix}\textit{Dividend.} \\ \textit{Divisor.}\end{matrix}\quad \frac{60}{15} = \frac{\cancel{3} \times \cancel{5} \times 4}{\cancel{3} \times \cancel{5}} = 4. \textit{ Quotient.}$$

EXPLANATION. Factoring the Divisor into 3 and 5, and the Dividend into 3, 5 and 4, and rejecting the Factors 3 and 5 from both, by Principle 8, we have 4 for the Quotient.

By Cancellation find the Quotients in the following

EXERCISES FOR THE SLATE AND BOARD.

$$\frac{48}{12},\quad \frac{70}{14},\quad \frac{168}{24},\quad \frac{210}{30},\quad \frac{385}{35},\quad \frac{735}{105},\quad \frac{1728}{144}.$$

When the Divisor is contained in the Dividend without a Remainder, the Divisor is named an ***Exact Divisor.***

EXAMPLE. Divide 30 by 42.

SOLUTION.

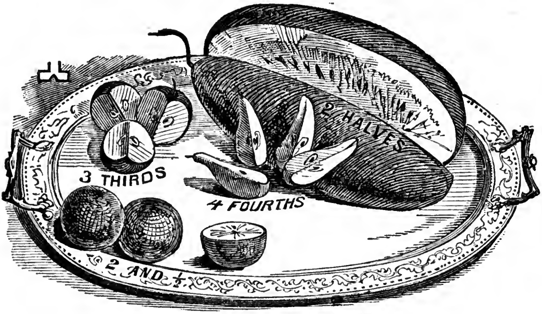
$$\begin{matrix}\textit{Dividend.} \\ \textit{Divisor.}\end{matrix}\quad \frac{30}{42} = \frac{\cancel{2} \times \cancel{3} \times 5}{\cancel{2} \times \cancel{3} \times 7} = \frac{5}{7}. \textit{ Ans.}$$

EXPLANATION. Factoring and canceling, we find no Factor 7 in the Dividend. Hence,

General Principle in Division.

9. *The Dividend cannot be exactly divided by the Divisor when any Factor of the Divisor is not found in it.*

FRACTIONS.



LESSON CII.

Example. Clifford's mother divided a watermelon between him and his sister. What did each receive?

Explanation. Writing the Dividend and Divisor as in former cases, we find that we can not so factor the Dividend as to obtain a Factor 2. Hence, according to Principle 9, 1 can not be exactly divided by 2.

SOLUTION.

Dividend. $\overline{1}$
Divisor. 2

But since there are *2 children*, and there is only *1 melon*, it is evident that the melon must be divided into *2 equal parts*, and *1 part* given to each child.

When any single thing is divided into *2 equal* parts, these parts are named ***Halves.*** One of these is called *one Half*, and is written $\frac{1}{2}$.

If an apple be cut into *3* equal parts, these parts are named ***Thirds.*** One part is named *one Third*, and written $\frac{1}{3}$. Two Thirds are written $\frac{2}{3}$.

If a pear be divided into *4* equal parts, these parts are named ***Fourths.*** One Fourth is written $\frac{1}{4}$; two Fourths are written $\frac{2}{4}$; and three Fourths $\frac{3}{4}$.

If *5 oranges* are to be divided between *2 children*, we can give each child *2 oranges*, that is 4 oranges to the 2 children, and then divide the fifth orange into 2 Halves, and give *1 Half-orange* to each child. Each child would then have *2 oranges and 1 Half-orange;* which are written *$2\frac{1}{2}$ oranges.*

When a watermelon is divided into 2 equal parts, or an apple into 3 equal parts, the melon or apple is cut or *fractured*, and one of the parts is a *fragment*, or ***Fraction*** of the entire thing. Hence, one Half, one Third, one Fourth, two Thirds, three Fourths, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$, are named *Fractions.*

DEFINITIONS.

1. *An **Integral Unit** is a single entire thing.*

2. *A **Fractional Unit** is one of the equal parts into which an Integral Unit is divided.*

3. *An **Integral Number,** or **Integer,** is an Integral Unit or collection of Integral Units.*

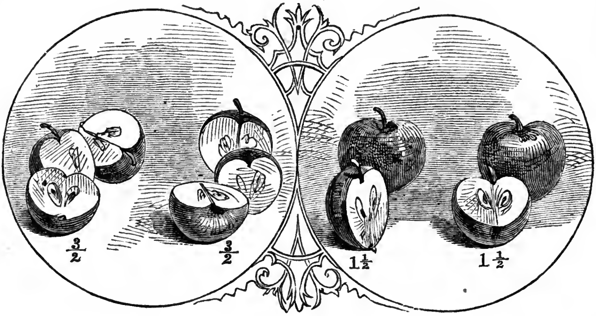
4. *A **Fractional Number,** or **Fraction,** is a Fractional Unit, or a collection of Fractional Units.*

5. *A **Mixed Number** is a number consisting of both an Integer and a Fraction.*

REMARKS.

1. *Integral* means *whole,* or *entire.* Hence, an *Integer* is frequently called a ***Whole Number.***

2. An Integral Unit is commonly called simply a *Unit;* or, sometimes, a ***Unit One.***



LESSON CIII.

Suppose we wish to divide *3* apples equally between *2* persons. 1st: It is evident that we can divide each apple into 2 equal parts, and give each person 1 part from each apple. He would have as many parts as there were apples; or 3 parts. He would receive 3 Halves; or $\frac{3}{2}$. 2d: If we chose, however, we might divide the 3 apples between the 2 persons by at first giving each 1 apple, or 2 apples to both, and then cutting the third apple into 2 equal parts and giving 1 part to each. Each person would thus have *1 entire apple*, and *1 Half-apple;* or *$1\frac{1}{2}$ apples.* It is plain that each person would receive the same in both cases; hence, $\frac{3}{2}$ are the same as $1\frac{1}{2}$. This must be evident. For, since 2 of the 3 Halves will make *one apple*, 3 Halves are the same as $1\frac{1}{2}$.

If 4 apples were to be divided among 3 persons, we might cut *each* apple into 3 pieces, and give each person a piece from *each* apple. Each person would then have 4 pieces, or $\frac{4}{3}$. And, since 3 Thirds make 1 apple, he would have the same as 1 apple and one Third of an apple; or $1\frac{1}{3}$ apples.

In the expressions, $\frac{3}{2}$ and $\frac{4}{3}$, the *Dividends,* 3 apples and 4 apples, show the *number of things divided;* and, since each person has 1 piece from each thing divided, 3 and 4 also show the *number of parts each person receives.* The *Divisors,* 2 and 3, show the *name* or kind *of the parts* received by each person. *Name* means the same as *Denomination.*

The Expressions $\frac{3}{2}$ and $\frac{4}{3}$ are Fractions. Hence,

DEFINITIONS.

6. *The* DIVIDEND *in a Fraction* is named the ***Numerator,*** *because it tells the* NUMBER *of parts in the Fraction.*

7. *The* DIVISOR *in a Fraction is named the* ***Denominator,*** *because it tells the* NAME, *or* DENOMINATION, *of the parts in the Fraction.*

8. *The* NUMERATOR *and* DENOMINATOR, *taken together, are named the* ***Terms*** *of the Fraction.*

9. *The* LINE (*Sign Minus*) *written between the Terms of a Fraction is named the* ***Dividing-line,*** *because it shows that the Numerator is* TO BE DIVIDED *by the Denominator.*

10. *The* ***Value*** *of a Fraction is the* QUOTIENT *arising from dividing the Numerator by the Denominator.*

11. *A* ***Minor Fraction*** *is a Fraction whose* VALUE *is* LESS THAN *the* UNIT ONE.

12. *A* ***Major Fraction*** *is a Fraction whose* VALUE EQUALS OR EXCEEDS *the* UNIT *One; that is, whose value is greater* than that of *any Minor Fraction.*

REMARK.

Minor means *less,* or *smaller;* and *Major* means *greater.*

13. ***Like Fractions*** *are Fractions having* LIKE OR EQUAL DENOMINATORS.

14. ***Unlike Fractions*** *are Fractions having* UNLIKE OR UNEQUAL DENOMINATORS.

REMARK.

$\frac{2}{7}$ and $\frac{4}{7}$ are Like Fractions, $\frac{3}{8}$ and $\frac{5}{11}$ Unlike.

15. *To* ***Reduce a Fraction*** *is to* CHANGE ITS FORM WITHOUT CHANGING ITS VALUE.

EXAMPLE 1. How many apples in $\frac{12}{2}$ apples?

EXPLANATION. Since *2 Halves* make *one* apple, we shall have *as many* apples as 2 Halves are contained *times* in 12 Halves; which are *6 times*. Hence $\frac{12}{2} = 6$.

EXAMPLE 2. How many apples in $\frac{16}{3}$ apples?

EXPLANATION. In 16 Thirds there are *as many* Ones as 3 Thirds are contained *times* in 16 Thirds. 3 are in 16 5 times, with 1 for a Remainder. Hence, 16 Thirds, or $\frac{16}{3}$, are equal to 5 Units and 1 Third; or $5\frac{1}{3}$. Hence,

TO REDUCE A MAJOR FRACTION TO AN INTEGER, OR MIXED NUMBER:

RULE.

Divide the Numerator of the Fraction by the Denominator; and if there is a Remainder use it for the Numerator of a Fraction, with the Divisor for Denominator, and annex this Fraction to the Quotient.

Reduce the Fractions to Integers or Mixed Numbers in these

EXERCISES FOR THE SLATE AND BOARD.

I.

$\frac{11}{2}$; $\frac{17}{5}$; $\frac{39}{8}$; $\frac{148}{6}$; $\frac{739}{7}$; $\frac{90}{9}$; $\frac{360}{6}$; $\frac{540}{90}$.

II.

$\frac{84}{21}$; $\frac{795}{25}$; $\frac{1189}{32}$; $\frac{1973}{176}$; $\frac{19000}{1700}$; $\frac{7354}{679}$.

LESSON CIV.

EXAMPLE 1. In 5 apples how many Thirds?

EXPLANATION. Since there are 3 Thirds in 1 apple, in 5 apples there are 5 times 3 Thirds; which are 15 Thirds; or $\frac{15}{3}$. Or, since there are 3 times as many Thirds as there are apples, we may find the number of Thirds by multiplying the number of apples by 3. 3 times 5 are 15. Hence, there are 15 Thirds; or $\frac{15}{3}$. Therefore,

TO REDUCE AN INTEGER TO THE FORM OF A FRACTION:

RULE.

Multiply the Integer by the Denominator of the required Fraction, and under this Product, used as the Numerator of the result, write the required Denominator.

EXERCISES FOR THE SLATE AND BOARD.

Reduce 7 to Thirds;	13 to Fifths;	37 to Ninths.
Reduce 123 to 25ths;	527 to 75ths;	317 to 11ths.

EXAMPLE 2. In $8\frac{2}{3}$ apples how many Thirds?

EXPLANATION. Reducing 8, or 8 apples, to Thirds, by the preceding Rule, we have (3 times 8 are) 24 Thirds. We have also 2 other Thirds ($\frac{2}{3}$). Adding 24 Thirds and 2 Thirds, we have for a result 26 Thirds; or $\frac{26}{3}$. Hence,

TO REDUCE A MIXED NUMBER TO THE FORM OF A FRACTION:

RULE.

Multiply the Integer by the Denominator of the Fraction, and to this Product add the Numerator. Under this Sum, used as a Numerator, write the Denominator of the given Fraction for a Denominator.

11

Exercises for the Slate and Board.

I.

$5\frac{3}{4}$; $11\frac{5}{7}$; $19\frac{3}{8}$; $23\frac{2}{9}$; $265\frac{7}{11}$; $187\frac{7}{15}$; $378\frac{11}{19}$.

II.

$183\frac{57}{65}$; $723\frac{73}{91}$; $618\frac{123}{575}$; $372\frac{852}{961}$; $958\frac{713}{912}$.

If James has 5 oranges and John 3 oranges, we find how many they both have by adding together 5 and 3 and obtaining their Sum, 8. So if they have things of any other kind, and the things which they both have are of the *same kind*, we find how many they both have by adding the numbers showing how many each has.

Example 3. Frank has $\frac{3}{8}$ of a watermelon, and Harry has $\frac{2}{8}$ of it. What have both?

Explanation. Since Frank had 3 pieces and Harry 2 pieces, and both had pieces of the *same kind*, or *size*, we add 3 pieces and 2 pieces, and have 5 pieces, or $\frac{5}{8}$. Hence,

To Add Like Fractions:

Rule.

Find the Sum of the Numerators of the Fractions, and, using this for the Numerator of the result, write the common Denominator for a Denominator.

Exercises for the Slate and Board.

I.

$\frac{5}{9}+\frac{3}{9}=?$ $\frac{8}{17}+\frac{7}{17}=?$ $\frac{15}{25}+\frac{13}{25}=?$ $\frac{11}{32}+\frac{27}{32}=?$

II.

$\frac{14}{35}+\frac{17}{35}=?$ $\frac{21}{56}+\frac{13}{56}=?$ $\frac{32}{85}+\frac{27}{85}=?$ $\frac{73}{125}+\frac{31}{125}=?$

Since Subtraction is the *reverse* of Addition, from our Rule for the Addition of Fractions we must have

TO SUBTRACT A FRACTION FROM A LIKE FRACTION:

RULE.

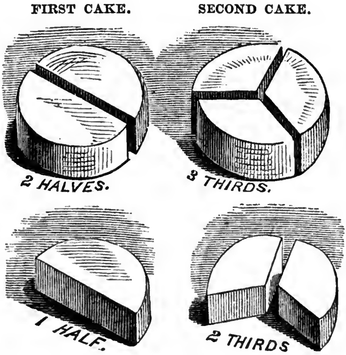
Subtract the Numerator of the Subtrahend from that of the Minuend, and, using the Difference as the Numerator of the result, write the common Denominator for a Denominator.

EXERCISES FOR THE SLATE AND BOARD.

$\frac{7}{8} - \frac{4}{8} = ?$ $\frac{11}{13} - \frac{7}{13} = ?$ $\frac{39}{27} - \frac{18}{27} = ?$ $\frac{49}{53} - \frac{37}{53} = ?$

LESSON CV.

EXAMPLE. Walter's mother gave him $\frac{1}{2}$ of a cake, and $\frac{2}{3}$ of another cake of the same size. What had he in all?



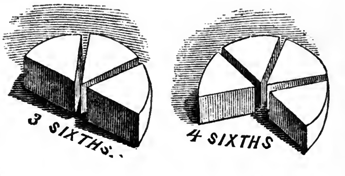
EXPLANATION. In the cut, at the right, we see the first cake cut into *Halves*, and the *Half* given to Walter placed below it.

We see also the second cake cut into *Thirds*, and the *2 Thirds* given to Walter placed below.

We observe that the 1 Half and the 2 Thirds *are not parts of the same kind, or size,* and hence *cannot be counted together, or added.* We must cut the 1 Half into *smaller parts, and*

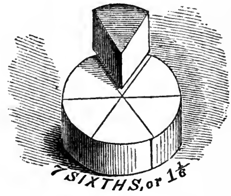
also the 2 Thirds into *smaller parts,* in such manner that *all the parts shall be of the same size.*

Cutting the 1 Half into 3 equal parts, as shown at the right, we see that there would be 6 such parts in the whole cake. Hence $\frac{1}{2}$ of the cake is the same as $\frac{3}{6}$ of the cake.



Cutting each of the 2 Thirds into 2 equal parts, they make 4 parts. There would be 6 such parts in the second cake. Hence the 4 parts are *Sixths ;* and it follows that $\frac{2}{3}$ of this cake are the same as $\frac{4}{6}$ of it.

Therefore Walter had $\frac{3}{6}$ of the first cake, and $\frac{4}{6}$ of the second cake. Since *these parts are all of the same size,* they can be added by the Rule in the last Lesson. Adding them according to the Rule, $\frac{3}{6}$ and $\frac{4}{6}$ are $\frac{7}{6}$. This is a Major Fraction. Reducing it to a Mixed Number according to the Rule on page 154, we have $1\frac{1}{6}$. This is shown in the cut at the right, by placing the 3 Sixths and 4 Sixths together.



6 of the 7 Sixths make a whole cake, and the remaining 1 Sixth is placed on the top of this cake. Hence Walter received 1 cake and 1 Sixth of a cake.

Thus we have first changed $\frac{1}{2}$ into $\frac{3}{6}$, and $\frac{2}{3}$ into $\frac{4}{6}$, and then added them and obtained $\frac{7}{6}$, or $1\frac{1}{6}$.

The *Numerator* of $\frac{1}{2}$ shows that there is *1 part in the Fraction ;* and the *Denominator,* 2, shows that there are 2 such parts in *one cake.* So, also, in *any* Fraction, the *Numerator* shows the *number of parts in the Fraction,*

and the *Denominator* shows the *number of such parts in a Unit.* Hence, when we cut the 1 part in the Numerator of $\frac{1}{2}$ into 3 parts, there will be *3 times as many such parts* as there are Halves in the cake; or 3 times 2 parts, which are 6 parts. Hence, cutting our $\frac{1}{2}$ cake into $\frac{3}{6}$, the *Numerator* and *Denominator* of $\frac{3}{6}$ are each *3 times as large* as the corresponding terms of $\frac{1}{2}$. That is, *we change $\frac{1}{2}$ to $\frac{3}{6}$ by multiplying both its terms by 3.* This *does not change the value* of the Fraction. This agrees with the 7th Principle of Division.

When we cut each of the 2 parts in $\frac{2}{3}$ into 2 parts, changing the 2 Thirds to 4 Sixths, or $\frac{2}{3}$ to $\frac{4}{6}$, we make both terms of $\frac{2}{3}$ *twice as large,* or *multiply both terms by 2.* This has *not changed the value* of the Fraction.

By multiplying both terms of each Fraction by the Denominator of the other, we have reduced the Unlike Fractions $\frac{1}{2}$ and $\frac{2}{3}$ to the Like Fractions $\frac{3}{6}$ and $\frac{4}{6}$.

In the same manner we may reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$, to Like Fractions by multiplying both terms of $\frac{1}{2}$ by the Denominators 3 and 4; both terms of $\frac{2}{3}$ by the Denominators 2 and 4; and both terms of $\frac{3}{4}$ by 2 and 3.

If we have any number of Unlike Fractions, we may proceed in the same manner. Hence,

To Reduce Unlike to Like Fractions:

Rule.

Multiply both terms of each Fraction by each of the other Denominators successively.

REMARK.—It is necessary to obtain the Denominator of only the first reduced Fraction, since all the other Denominators are like it.

Reduce Unlike to Like Fractions in the following

EXERCISES FOR THE SLATE AND BOARD.

I.

$\frac{2}{3}$ and $\frac{3}{4}$; $\frac{3}{5}$ and $\frac{4}{7}$; $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{6}{7}$; $\frac{3}{5}$, $\frac{4}{7}$ and $\frac{8}{11}$.

II.

$\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$ and $\frac{4}{11}$; $\frac{4}{5}$, $\frac{6}{7}$ and $\frac{9}{13}$; $\frac{2}{3}$, $\frac{5}{11}$, $\frac{10}{11}$ and $\frac{11}{17}$.

LESSON CVI.

EXAMPLE. Add $5\frac{2}{3}$ and $4\frac{6}{7}$.

EXPLANATION. Reducing $\frac{2}{3}$ and $\frac{6}{7}$ to Like Fractions, we have $\frac{14}{21}$ and $\frac{18}{21}$. Adding these, we have $\frac{32}{21}$, or $1\frac{11}{21}$. Having the Sum of the Fractions, we add to this the Integers 5 and 4. The Sum of 5, 4 and 1 is 10. Writing the Fractional part of the Sum after this, we have $10\frac{11}{21}$. Hence,

SOLUTION.

$\frac{2}{3} = \frac{14}{21}$, and $\frac{6}{7} = \frac{18}{21}$.
$\frac{14}{21} + \frac{18}{21} = \frac{32}{21}$, or $1\frac{11}{21}$.
$5+4+1=10$. Hence
$5\frac{2}{3}$ and $4\frac{6}{7} = 10\frac{11}{21}$.

TO ADD MIXED NUMBERS:

RULE.

Add the Fractions, and if their Sum is a Major Fraction reduce it to an Integer or Mixed Number. Add the Integral part of this result with the given Integers, and to this *Sum annex the Fractional part.*

EXERCISES FOR THE SLATE AND BOARD.

I.

$2\frac{2}{5}+3\frac{4}{5}=?$ $7\frac{5}{8}+11\frac{7}{8}=?$ $12\frac{11}{15}+9\frac{13}{15}=?$ $1\frac{19}{21}+2\frac{2}{21}=?$

II.

$4\frac{2}{3}+5\frac{4}{5}=?$ $6\frac{5}{7}+8\frac{7}{9}=?$ $10\frac{1}{3}+9\frac{5}{8}=?$ $16\frac{5}{7}+13\frac{8}{11}=?$

EXAMPLE. From $8\frac{1}{3}$ subtract $4\frac{2}{5}$.

SOLUTION.

$\frac{1}{3}=\frac{5}{15}$, and $\frac{2}{5}=\frac{6}{15}$
$8\frac{5}{15}=7\frac{20}{15}$. Hence
$8\frac{1}{3}-4\frac{2}{5}=7\frac{20}{15}-4\frac{6}{15}$.
$\frac{20}{15}-\frac{6}{15}=\frac{14}{15}$, and $7-4=3$.
Hence $8\frac{1}{3}-4\frac{2}{5}=3\frac{14}{15}$.

EXPLANATION. Reducing $\frac{1}{3}$ and $\frac{2}{5}$ to the Like Fractions $\frac{5}{15}$ and $\frac{6}{15}$, we find that $\frac{6}{15}$ cannot be subtracted from $\frac{5}{15}$, since it exceeds it. Therefore we take one of the 8 Units, and, calling it $\frac{15}{15}$, add it to the $\frac{5}{15}$, and have $\frac{20}{15}$. Our Minuend is then $7\frac{20}{15}$, and our Subtrahend $4\frac{6}{15}$. Subtracting $\frac{6}{15}$ from $\frac{20}{15}$ we have $\frac{14}{15}$ for the Fractional part of our Remainder. Subtracting 4 Ones from 7 Ones, we have 3 Ones left. Uniting both parts of our Remainder, we have $3\frac{14}{15}$ for the true Remainder. Hence,

TO SUBTRACT A MIXED NUMBER, OR A FRACTION, FROM A MIXED NUMBER OR AN INTEGER:

RULE.

I. *Reduce to Like Fractions the Fraction in the Subtrahend, and also such part of the Minuend, (including the Fraction, if any,) as shall equal or exceed this.*

II. *Subtract the Fractional part of the Subtrahend from that in the Minuend, and the Integer in the Subtrahend from that in the Minuend, and unite the two partial Remainders into one Final Remainder.*

EXERCISES FOR THE SLATE AND BOARD.

I.

$7\frac{6}{8}-5\frac{3}{8}=?$ $\quad 9\frac{4}{5}-5\frac{1}{5}=?$ $\quad 11\frac{2}{3}-5\frac{8}{11}=?$ $\quad 10\frac{2}{5}-3\frac{5}{7}=?$

II.

$12\frac{1}{3}-5\frac{3}{7}=?$ $\quad 15-6\frac{4}{5}=?$ $\quad 21\frac{4}{11}-3\frac{1}{2}=?$ $\quad 35-21\frac{8}{13}=?$

LESSON CVII.

MULTIPLICATION AND DIVISION OF FRACTIONS.

In a Fraction the Numerator is a Dividend, and the Denominator a Divisor. The Value of the Fraction is the Quotient. Hence we may make any change, in the Fraction, which does not change the Quotient. But, according to the 8th Principle in Division, dividing both Dividend and Divisor (Numerator and Denominator) by the same number does not alter the Quotient.

If we take the Fraction $\frac{9}{12}$ and factor both terms, we have $\frac{3\times3}{3\times4}$. Dividing both terms by 3, by rejecting the common Factor 3, we have $\frac{3}{4}$. Hence $\frac{9}{12}$ equal $\frac{3}{4}$. The terms of $\frac{3}{4}$ are *smaller* than those of $\frac{9}{12}$, and hence more convenient. We reduced $\frac{9}{12}$ to $\frac{3}{4}$ by *rejecting the Factors common to both terms.*

DEFINITION.

A Fraction is expressed in its **Lowest Terms** *when there is no Factor common to both terms.* Hence,

To Reduce a Fraction to Lowest Terms:

Rule.

Reject all the Common *Factors from both Terms.*

Exercises for the Slate and Board.

$\frac{18}{24}$; $\frac{21}{27}$; $\frac{24}{36}$; $\frac{90}{150}$; $\frac{135}{189}$; $\frac{216}{360}$; $\frac{375}{600}$; $\frac{144}{1728}$.

Example. Multiply $\frac{1}{6}$ by 3.

Explanation. 1st. According to the 3d Principle in Division, multiplying the Dividend multiplies the Quotient. Hence,

SOLUTION.

First Method.

$\frac{1}{6}\times3=\frac{1\times3}{6}=\frac{3}{6}=\frac{1}{2}$. *Ans.*

Second Method.

$\frac{1}{6}\times3=\frac{1}{6\div3}=\frac{1}{2}$. *Ans.*

we multiply the Numerator of $\frac{1}{6}$ by 3, and obtain $\frac{3}{6}$ for a Product; or, in Lowest Terms, $\frac{1}{2}$.

2d. According to the 6th Principle in Division, dividing the Divisor multiplies the Quotient. Hence we multiply $\frac{1}{6}$ by 3 by dividing the Denominator by 3, and have $\frac{1}{2}$ for our Final Product, as before. Hence,

To Multiply a Fraction by an Integer:

Rule.

Multiply the Numerator of the Fraction by the Integer, and for a Denominator write the given Denominator; or, Divide the Denominator of the Fraction by the Integer, and write the given Numerator over this for a Numerator.

Exercises for the Slate and Board.

First Method.

$\frac{1}{5} \times 3 = ?$ $\frac{3}{7} \times 2 = ?$ $\frac{2}{11} \times 4 = ?$ $\frac{3}{37} \times 11 = ?$

Second Method.

$\frac{2}{9} \times 3 = ?$ $\frac{10}{105} \times 7 = ?$ $\frac{2}{45} \times 9 = ?$ $\frac{6}{56} \times 8 = ?$

Example. Divide $\frac{6}{7}$ by 3.

Explanation. According to the 4th General Principle in Division, dividing the Dividend divides the Quotient.

SOLUTION.

First Method.

$\frac{6}{7} \div 3 = \frac{6 \div 3}{7} = \frac{2}{7}$. *Quotient.*

Hence, we divide the Numerator of $\frac{6}{7}$ by 3, and obtain $\frac{2}{7}$ for our Quotient.

According to the 5th General Principle in Division, multiplying the Divisor divides the Quotient. Hence, multiplying the Denominator of $\frac{6}{7}$ by 3, we have $\frac{6}{21}$; or, in Lowest Terms, $\frac{2}{7}$, as before. Hence,

Second Method.

$\frac{6}{7} \div 3 = \frac{6}{7 \times 3} = \frac{6}{21} = \frac{2}{7}$. *Quotient.*

To Divide a Fraction by an Integer:

Rule.

Divide the Numerator of the Fraction by the Integer, and under this Quotient write the given Denominator for a Denominator; or, *Multiply the Denominator of the Fraction by the Integer, and over this Product write the given Numerator for a Numerator.*

Remark. The result obtained by the Second Method should be reduced to Lowest Terms.

Exercises for the Slate and Board.

First Method.

$\frac{9}{11} \div 3 = ?$ $\frac{15}{17} \div 5 = ?$ $\frac{24}{25} \div 8 = ?$ $\frac{24}{29} \div 12 = ?$

Second Method.

$\frac{1}{2} \div 3 = ?$ $\frac{5}{7} \div 6 = ?$ $\frac{3}{30} \div 5 = ?$ $\frac{6}{126} \div 7 = ?$

LESSON CVIII.

Example 1. Multiply 14 by $\frac{1}{7}$.

Explanation To multiply 14 by $\frac{1}{7}$ is to take one-seventh of 14.

SOLUTION.

$14 \times \frac{1}{7} = 14 \div 7 = \frac{14}{7} = 2$. *Prod.*

To take one-seventh of 14 we must divide 14 by 7. The result, written as a Fraction, is $\frac{14}{7}$.

Example 2. Multiply 14 by $\frac{3}{7}$.

SOLUTION.

$\frac{3}{7} = \frac{1}{7} \times 3.$

$14 \times 3 = 42.$

$42 \times \frac{1}{7} = 42 \div 7 = \frac{42}{7} = 6$. *Product.*

Explanation. We have seen that when our Multiplier is composed of Factors we can obtain the Product by multiplying by the several Factors in succession. Finding that our Multiplier is composed of two Factors, we

obtain our Product by multiplying by its Factors, 3 and $\frac{1}{7}$. 14 × 3 gives 42. And 42 × $\frac{1}{7}$ is the same as 42 ÷ 7, which is $\frac{42}{7}$, or 6.

EXAMPLE 3. Multiply $\frac{4}{5}$ by $\frac{3}{7}$.

SOLUTION.

$$\frac{3}{7}=\frac{1}{7}\times 3$$

$$\frac{4}{5}\times 3=\frac{4\times 3}{5}=\frac{12}{5}$$

$$\frac{12}{5}\times\frac{1}{7}=\frac{12}{5}\div 7=\frac{12}{5\times 7}=\frac{12}{35}.\ \textit{Prod.}$$

EXPLANATION. We multiply $\frac{4}{5}$ by 3 and $\frac{1}{7}$, the Factors of $\frac{3}{7}$. We multiply $\frac{4}{5}$ by 3 by multiplying the Numerator by 3. We multiply this result by $\frac{1}{7}$ by dividing by 7; which we do by multiplying the Denominator by 7. This gives $\frac{12}{35}$ for the Product. Examining these Examples and Solutions, we see that we multiply by a Fraction by multiplying the Multiplicand by the Numerator of the Multiplier, and dividing the result by the Denominator of the Multiplier. Hence,

To Multiply by a Fraction:

Rule.

Multiply the Multiplicand by the Numerator of the Multiplier, and divide this result by the Denominator.

Remarks.

1. Mixed Numbers can be multiplied together by the above Rule, after reducing them to Major Fractions.
2. The Product should be reduced to Lowest Terms, or to an Integer or Mixed Number.

Exercises for the Slate and Board.

Fractions.

$8\times\frac{1}{9}=?$	$12\times\frac{3}{4}=?$	$15\times\frac{3}{5}=?$	$21\times\frac{5}{7}=?$
$\frac{2}{3}\times\frac{4}{5}=?$	$\frac{5}{7}\times\frac{3}{8}=?$	$\frac{9}{11}\times\frac{7}{8}=?$	$\frac{11}{15}\times\frac{7}{9}=?$

Mixed Numbers.

$2\frac{1}{3}\times 1\frac{3}{4}=?$	$5\frac{2}{7}\times 3\frac{4}{5}=?$	$7\frac{5}{6}\times 8\frac{2}{3}=?$	$9\frac{5}{11}\times 3\frac{6}{7}=?$

LESSON CIX.

EXAMPLE 1. Divide 5 by $\frac{1}{7}$.

EXPLANATION. It is plain that $\frac{1}{7}$ is contained in 1 seven times. It must be contained in 2 *twice* 7 times, and in 5 *five* times 7 times, or, which is the same, 7 times 5 times; which are 35 times. Hence we divide 5 by $\frac{1}{7}$ by multiplying 5 by 7.

SOLUTION.

$5 \div \frac{1}{7} = 5 \times 7 = 35$. *Quo't.*

EXAMPLE 2. Divide 5 by $\frac{3}{7}$.

EXPLANATION. We factor our Divisor $\frac{3}{7}$ into $\frac{1}{7}$ and 3. Dividing first by the Factor 3, we have $\frac{5}{3}$. Dividing this result by the other Factor, $\frac{1}{7}$, by multiplying by 7, we have $\frac{35}{3}$ for the result, or Quotient.

SOLUTION.

$\frac{3}{7} = \frac{1}{7} \times 3$

$5 \div 3 = \frac{5}{3}$

$\frac{5}{3} \div \frac{1}{7} = \frac{5}{3} \times 7 = \frac{5 \times 7}{3} = \frac{35}{3}$. *Quo't.*

EXAMPLE 3. Divide $\frac{3}{4}$ by $\frac{5}{7}$.

EXPLANATION. Factoring $\frac{5}{7}$ into $\frac{1}{7}$ and 5, we divide $\frac{3}{4}$ by 5, by multiplying the Denominator by 5, and have for a result $\frac{3}{20}$. We then divide $\frac{3}{20}$ by $\frac{1}{7}$, by multiplying by 7. This gives $\frac{21}{20}$ for the Quotient. Reducing $\frac{21}{20}$ to a Mixed Number, we have $1\frac{1}{20}$ for our final Quotient.

SOLUTION.

$\frac{5}{7} = \frac{1}{7} \times 5$

$\frac{3}{4} \div 5 = \frac{3}{4 \times 5} = \frac{3}{20}$

$\frac{3}{20} \div \frac{1}{7} = \frac{3}{20} \times 7 = \frac{3 \times 7}{20} = \frac{21}{20}$

$\frac{21}{20} = 1\frac{1}{20}$. *Ans.*

Examining these Solutions and Explanations, we see that we have in each case divided by a Fraction by dividing the Dividend by the Numerator of the Divisor, and then multiplying this result by the Denominator of the Divisor. Hence,

To Divide by a Fraction:

Rule.

Divide the Dividend by the Numerator of the Divisor, and then multiply this result by the Denominator of the Divisor.

Remarks.

1. The result should be reduced to Lowest Terms; if a Major Fraction, to an Integer or Mixed Number.

2. Mixed Numbers in the Dividend or Divisor may be reduced to Major Fractions, and then the Division be performed by the above Rule.

Exercises for the Slate and Board.

Fractions.

$9 \div \frac{2}{3} = ?$ $8 \div \frac{4}{5} = ?$ $15 \div \frac{5}{7} = ?$ $1 \div \frac{2}{3} = ?$

$\frac{3}{4} \div \frac{2}{3} = ?$ $\frac{7}{8} \div \frac{5}{9} = ?$ $\frac{8}{11} \div \frac{3}{4} = ?$ $\frac{24}{25} \div \frac{8}{15} = ?$

Mixed Numbers.

$2\frac{1}{2} \div 3\frac{2}{5} = ?$ $5\frac{3}{4} \div 7\frac{5}{9} = ?$ $12\frac{3}{7} \div 4\frac{1}{7} = ?$ $3\frac{1}{2} \div 1\frac{3}{4} = ?$

To Teachers.—The Elementary Principles involved in the Reduction of Denominate Numbers, the Addition and Subtraction of Compound Numbers, and the Multiplication and Division of Compound by Simple Numbers, are the same which are used in Addition, Subtraction, Multiplication and Division of Simple Numbers, and have already been fully set forth. In applying them to Denominate and Compound Numbers, the slight changes necessary to be made can be readily and easily explained by Teachers. Therefore, the Examples hereinafter given are not accompanied with explanations.

DENOMINATE NUMBERS.

LESSON CX.

In *measuring* a quantity, we take some *definite amount* of it for a *Unit;* as a pint, a yard. We often have *different Units* for the *same kind* of quantity.

DEFINITIONS.

1. ***A Unit of Measure** is the definite amount of anything taken as a* STANDARD OF COMPARISON *in measuring all quantities of that kind.*

2. ***Denomination** is the* NAME *given to a Unit of Measure; as quart, ounce, shilling.*

3. ***A Higher Denomination** is that one of two Denominations whose Unit has the* HIGHER VALUE.

4. ***A Lower Denomination** is that one of two Denominations whose Unit has the* LOWER VALUE.

5. ***A Denominate Number** is a number applied to one or more Denominations; as 4 gallons, 3 quarts.*

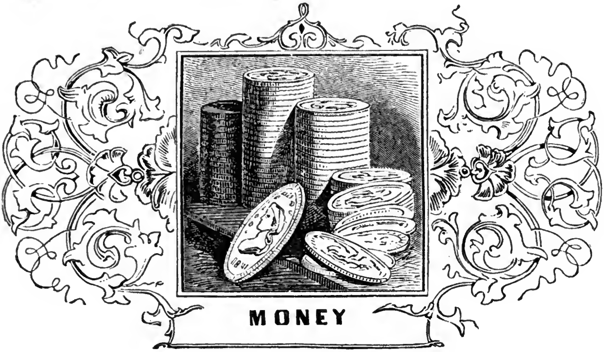
6. ***A Simple Number** is a number expressed either in* NO *Denomination or only* ONE.

7. ***A Compound Number** is a number expressed in* MORE THAN ONE DENOMINATION; *as 2 days 5 hours.*

8. ***Reduction of Denominate Numbers** is changing the* NUMBER *and* DENOMINATION *of a Denominate Number without changing its* VALUE.

9. ***Reduction Ascending** is reducing a Denominate Number to one of a* HIGHER DENOMINATION.

10. ***Reduction Descending** is reducing a Denominate Number to one of a* LOWER DENOMINATION.



LESSON CXI.

UNITED STATES MONEY.

United States Money, called also ***Federal Money,*** is the legal currency of the United States.

TABLE.

10 mills (*m*).	are 1 cent.	*ct.*
10 cents	are 1 dime.	*d.*
10 dimes, or 100 ct.,	are 1 dollar.	*$.*
10 dollars	are 1 eagle.	*E.*

NOTE.—The Table of Canada Money is the same as that of United States Money.

Exercises in Reduction.

In 9 ct. how many mills?
In 8 ct. 7 m. how many mills?
In 7 d. how many cents? How many mills?
In 7 d. 8 ct. 9 m. how many mills?
How many cents in 80 m.? In 50 m.? In 90 m.?
How many cents and mills in 98 m.? In 65 m.?

Addition.

What is the Sum of 4 ct. 5 m., and 3 ct. 2 m.?
What is the Sum of 6 ct. 8 m., and 5 ct. 7 m.?
What is the Sum of 8 d. 9 ct. 7 m., and 3 d. 8 ct. 6 m.?

Subtraction.

From 8 ct. 9 m. subtract 5 ct. 3 m.?
From 7 d. 8 ct. 9 m. subtract 3d. 4 ct. 5 m.
From 5 d. 4 ct. 7 m. subtract 2 d. 9 ct. 3 m.
From 8 d. 3 ct. 4 m. subtract 3 d. 5 ct. 7 m.

LESSON CXII.

ENGLISH MONEY.

English Money, called also ***Sterling Money,*** is the currency of Great Britain.

TABLE.

4 farthings (*far.*)	are 1 penny.	*d.*
12 pence	are 1 shilling.	*s.*
20 shillings	are 1 pound or sovereign.	£.
21 shillings	are 1 guinea.	*guin.*

Exercises in Reduction.

How many farthings in 7d.? In 9s.? In 3s. 9d.?
How many pence in 7s.? In £3? In £5 7s. 9d.?
How many farthings in £7 11s. 5d.? In £9 7s. 8d. 3 far.?
How many shillings in 48d.? In 96 far.? In 240 far.?
Reduce 987 far. to Higher Denominations.
Reduce 11s. and 1,765 far. to Higher Denominations.
In 21 pounds how many guineas?

LESSON CXIII.

Liquid Measure, called also ***Wine Measure,*** is used in measuring wines, oil, molasses, milk, and other liquids.

TABLE.

4	gills (*gi.*)	are 1 pint.	*pt.*
2	pints	are 1 quart.	*qt.*
4	quarts	are 1 gallon.	*gal.*
$31\frac{1}{2}$	gallons	are 1 barrel.	*bbl.*
2	barrels, or	are 1 hogshead.	*hhd.*
63	gallons,		

EXERCISES FOR THE SLATE AND BOARD.

Reduction.

How many gills in 5 pt.? In 7 qt.? In 15 gal.?
How many gills in 15 gal. 3 qt. 1 pt. 2 gi.?
Reduce 547 gi. to Higher Denominations.

Addition.

To 19 gal. 3 qt. 1 pt. 3 gi. add 9 gal. 2 qt. 1 pt. 2 gi.?
To 20 gal. 2 qt. 1 pt. 2 gi. add 10 gal. 3 qt. 1 pt. 3 gi.?

Subtraction.

From 27 gal. 3 qt. 1 pt. 3 gi. take 19 gal. 2 qt. 1 pt. 2 gi.

From 9 gal. 0 qt. 1 pt. 1 gi. take 2 gal. 2 qt. 0 pt. 3 gi.

Multiplication.

How much oil is there in 5 casks, each containing 12 gal. 2 qt. 1 pt. 3 gi.?

In 3 casks, each containing 10 gal. 1 qt. 1 pt.?

Division.

If 18 gal. 3 qt. 1 pt. 2 gi. of milk be equally divided between two persons, what will each receive?

LESSON CXIV.

Dry Measure is used in measuring grain, fruits, roots, seeds, salt, lime, charcoal, and various other articles not fluid.

TABLE.

2 pints (*pt.*)	are 1 quart.	*qt.*
8 quarts	are 1 peck.	*pk.*
4 pecks	are 1 bushel.	*bu.*
36 bushels (of coal)	are 1 chaldron.	*chal.*

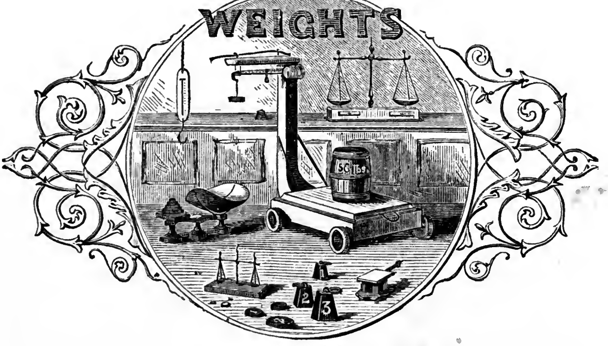
Exercises for the Slate and Board.

Addition.

Add 5 bu. 3 pk. 7 qt. 1 pt. and 3 bu. 2 pk. 4 qt. 1 pt.
Add 7 bu. 1 pk. 5 qt. 1 pt. and 11 bu. 3 pk. 7 qt. 1 pt.

Subtraction.

From 31 bu. 3 pk. 7 qt. 0 pt. take 27 bu. 3 pk. 4 qt. 1 pt.
From 25 bu. 3 pk. 0 qt. 0 pt. take 16 bu. 2 pk. 5 qt. 1 pt.



LESSON CXV.

AVOIRDUPOIS WEIGHT.

Avoirdupois Weight is used for all the ordinary purposes of weighing.

TABLE.

16 drams (*dr.*)	are 1	ounce.	*oz.*
16 ounces	are 1	pound.	*lb.*
25 pounds	are 1	quarter.	*qr.*
4 quarters, or 100 pounds,	are 1	hundred-weight.	*cwt.*
20 hundred weight	are 1	ton.	*T.*

Exercises for the Slate and Board.

Reduction.

How many drams in 3 qr. 18 lb. 13 oz. 11 dr.?
Reduce 1572 dr. to Higher Denominations.

Addition.

To 1 qr. 17 lb. 9 oz. 7 dr. add 1 qr. 15 lb. 13 oz. 14 dr.
To 3 qr. 21 lb. 11 oz. 13 dr. add 2 qr. 9 lb. 8 oz. 7 dr.

LESSON CXVI.

TROY WEIGHT.

Troy Weight is used in weighing jewels, gold and silver.

TABLE.

24 grains (*gr.*)	are 1 pennyweight.	*pwt.*
20 pennyweights	are 1 ounce.	*oz.*
12 ounces	are 1 pound.	*lb.*

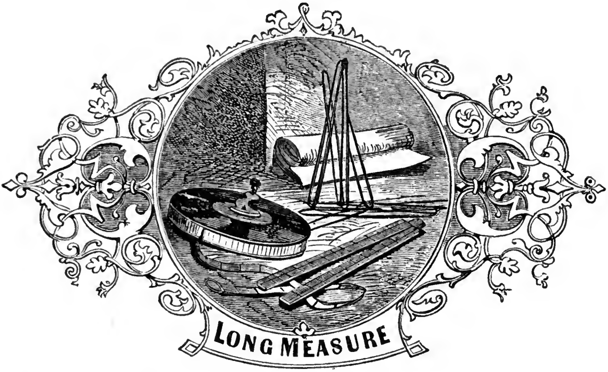
NOTE. The Teacher will supply under this and the following Tables all needed Exercises.

APOTHECARIES' WEIGHT.

Apothecaries' Weight is used by physicians in compounding medicines; but when medicines are bought or sold Avoirdupois Weight is used.

TABLE.

20 grains (*gr.*)	are 1 scruple.	*sc.*	or	℈.
3 scruples	are 1 dram.	*dr.*	or	ʒ.
8 drams	are 1 ounce.	*oz.*	or	℥.
12 ounces	are 1 pound.	*lb.*	or	℔.



LESSON CXVII.

LINEAR MEASURE.

Linear Measure—called also ***Long Measure*** —is used in measuring lines, or distances.

TABLE.

12	inches (*in.*)	are 1 foot.	*ft.*
3	feet	are 1 yard.	*yd.*
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ ft.,	are 1 rod, perch, or pole.	*rd.*
40	rods	are 1 furlong.	*fur.*
8	furlongs, or 320 rd.,	are 1 mile.	*mi.*

SQUARE MEASURE.

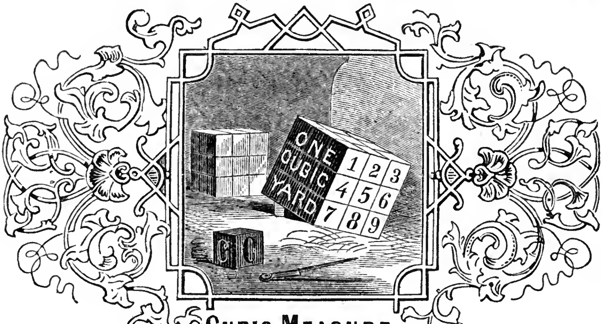
Square Measure is used for measuring surfaces; as of land, plastering, and paving.

A square foot is a square each of whose 4 sides is 1 foot, or 12 inches, in length. A square yard is a square each of whose 4 sides is 1 yard, or 3 feet, in length.

TABLE.

144	square inches (*sq. in.*)	are 1 square foot.	*sq. ft.*
9	square feet	are 1 square yard.	*sq. yd.*
$30\frac{1}{4}$	square yards	are 1 square rod.	*sq. rd.*
40	square rods	are 1 rood.	*R.*
4	roods, or	are 1 acre.	*A.*
160	sq. rods,		
640	acres	are 1 square mile.	*sq. mi.*

LESSON CXVIII.



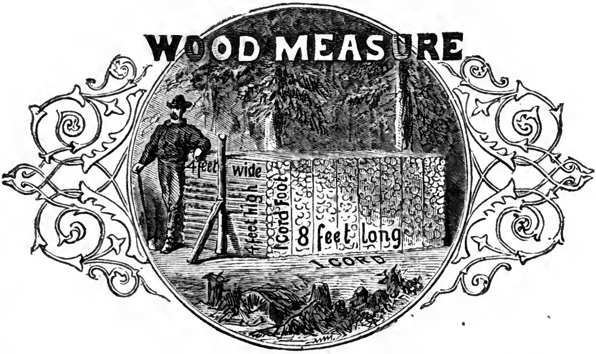
CUBIC MEASURE

Cubic Measure is used for measuring solids; as timber, wood, and stone.

A cubic foot is a cube each of whose 12 edges is 1 foot, or 12 inches, in length. A cubic yard is a cube measuring 1 yard, or 3 feet, on each edge. Each of its 6 faces is a square containing 9 square feet. A cubic yard is shown in the above cut.

TABLE.

1728 cubic inches	are 1 cubic foot.	*cu. ft.*
27 cubic feet	are 1 cubic yard.	*cu. yd.*
42 cubic feet	are 1 ton, shipping.	*t. s.*
$24\frac{3}{4}$ cubic feet	are 1 perch, of stone.	*pch.*



Wood Measure, though part of Cubic Measure, is sometimes embraced in a separate Table.

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, contains a cord.

TABLE.

16 cubic feet	are 1 cord foot.	*cd. ft.*
8 cord feet, or 128 cubic feet,	are 1 cord.	*cd.*

EXERCISES FOR THE SLATE AND BOARD.

In 1 cd. 5 cd. ft. 11 cu. ft. 187 cu. in., how many cubic inches?

Reduce 3 cu. yd. 5 cu. ft. 127 cu. in. to cubic inches.

To 3 cd. 7 cd. ft. 11 cu. ft. add 5 cd. 9 cd. ft. 8 cu. ft.

LESSON CXIX.

TIME MEASURE.

Time is *Duration* having a beginning and an end Being definite, it can therefore be measured.

The Day and Year are the Natural Divisions of Time, since they are founded in Nature.

TABLE.

60 seconds (*sec.*)	are 1 minute.	*min.*
60 minutes	are 1 hour.	*h.*
24 hours	are 1 day.	*da.*
7 days	are 1 week.	*wk.*
365 days, or 52 wk. 1 d.,	are 1 common year.	*yr.*
366 days, or 52 wk. 2 d.,	are 1 leap year.	*leap yr.*
12 calendar months	are 1 year.	*yr.*
100 years	are 1 century.	*C.*

Every fourth year in a century (except sometimes the last) is a leap year; as 1804, 1808, 1812.

DIVISION OF THE YEAR.

	Months.		Days.	Seasons.
January,	Jan.,	1st month,	has 31.	Winter.
February,	Feb.,	2d month,	has 28 or 29.	
March,	Mar.,	3d month,	has 31.	Spring.
April,	Apr.,	4th month,	has 30.	
May,	May,	5th month,	has 31.	
June,	June,	6th month,	has 30.	Summer.
July,	July,	7th month,	has 31.	
August,	Aug.	8th month,	has 31.	
September,	Sept.,	9th month,	has 30.	Autumn.
October,	Oct.,	10th month,	has 31.	
November,	Nov.,	11th month,	has 30.	
December,	Dec.,	12th month,	has 31.	Winter.

NOTE.—February has 29 days in none but leap years.

LESSON CXX.

CIRCULAR AND ANGULAR MEASURE.

Circular and ***Angular Measure*** is used in measuring angles, and in the comparison of portions of the circumferences of circles.

TABLE.

60 seconds (″)	are	1 minute.	′
60 minutes	are	1 degree.	°
30 degrees	are	1 sign.	*sig.*
90 degrees	are	1 quadrant, or right angle.	*quad.* *r. a.*
360 degrees, or 12 signs, or 4 quadrants,	are	1 circumference of a circle.	*cir.*